

**DEPARTMENT OF MATHEMATICS**  
**JADAVPUR UNIVERSITY**  
**M.Sc. Syllabus ( Semester System)**

1<sup>st</sup> Year M.Sc. 1<sup>st</sup> Semester

Unit 1.1	Algebra - 1
Unit 1.2	Real Analysis
Unit 1.3	Complex Analysis
Unit 1.4	General Mechanics
Unit 1.5	Differential Geometry

1<sup>st</sup> Year M.Sc. 2<sup>nd</sup> Semester

Unit 2.1	Algebra - II
Unit 2.2	Topology
Unit 2.3	Functional Analysis
Unit 2.4	Mechanics of Continua
Unit 2.5	Ordinary Differential Equation & Special Function

2<sup>nd</sup> year M.Sc. 1<sup>st</sup> Semester

Unit 3.1	Numerical Analysis Theory , 30 Marks Numerical Analysis Practical , 20 Marks Or Advanced Topology (50 Marks) Or Discrete Mathematics I (50 Marks)
Unit 3.2	Partial Differential Equation, 30 Marks Non Linear Ordinary Differential Equation , 20 Marks
Unit 3.3	Special Paper ( OP – A-1.1 to A-1.8)
Unit 3.4	Special Paper (OP - B-1.1 to B-1.39)
Unit 3.5	Special Paper (OP – B-1.1 to B-1.39 )

2<sup>nd</sup> Year M.Sc. 2<sup>nd</sup> Semester

Unit 4.1	Advanced Numerical Analysis Theory : 20 Marks Advanced Numerical Analysis Practical : 30 Marks Or Advanced Functional Analysis (50 Marks) Or Discrete Mathematics II (50 Marks)
Unit 4.2	Integral Equation 25 Integral Transforms 25
Unit 4.3	Special Paper (OP – A-2.1 to A-2.8 )
Unit 4.4	Special Paper (OP - B-2.1 to B-2.39)
Unit 4.5	Special Paper ( OP – B-2.1 to B-2.39)

# M.Sc. First Year First Semester (250 marks)

## Unit 1.1: Algebra-I (50 Marks)

### Groups (25 Marks) :

Homomorphism of groups, Normal Subgroups, Quotient Groups, Isomorphism Theorems, Cayley's Theorem.

Generalized Cayley's Theorem, Cauchy's Theorem, Group Action, Sylow Theorems and their applications.

Normal and Subnormal Series, Composition Series, Solvable Groups and Nilpotent Groups, Jordan-Hölder Theorem and its applications.

**Rings (15 Marks) :** Ideals and Homomorphisms, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Polynomial and Power Series Rings.

Divisibility Theory : Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss' Theorem.

Noetherian and Artinian Rings, Hilbert Basis Theorem, Cohen's Theorem.

**Modules (10 Marks) :** Left and Right Modules over a ring with identity, Cyclic Modules, Free Modules, Fundamental Structure Theorem for finitely generated modules over a PID and its applications to finitely generated abelian groups.

### References :

Dummit, D.S., Foote, R.M., *Abstract Algebra*, Second Edition, John Wiley & Sons, Inc., 1999.

Goldhaber, J.K., Ehrlich, G., *Algebra*, The Macmillan Company, Collier-Macmillan Limited, London.

Herstein, I.N., *Topics in Abstract Algebra*, Wiley Eastern Limited.

Hungerford, T.W., *Algebra*, Springer.

Jacobson, N., *Basic Algebra, I & II*, Hindusthan Publishing Corporation, India.

Malik, D.S., Mordesen, J.M., Sen, M.K., *Fundamentals of Abstract Algebra*, The McGraw-Hill Companies, Inc.

Rotman, J.J., *The Theory of Groups: An Introduction*, Allyn and Bacon, Inc., Boston.

## Unit 1.2 : Real Analysis (50 Marks)

### Fourier Series and Fourier Transformation .

#### Bounded Variation .

Functions of Bounded Variation and their properties, Differentiation of a function of bounded variation, Absolutely Continuous Function, Representation of an absolutely continuous function by an integral.

#### The Theory of Measure .

Semiring and ring of sets,  $\sigma$ -ring and  $\sigma$ -algebra, Ring and  $\sigma$ -ring generated by a class of sets, Monotone class of sets, Monotone class generated by a ring, Borel Sets, Measures on semirings and their properties, Outer Measure and Measurable Sets, Caratheodory Extension : Outer measure generated by a measure, Lebesgue measure on  $\mathbb{R}^n$ , Measure space, Finite and  $\sigma$ -finite measure spaces. Measurable Functions, Sequence of measurable functions, Egorov's Theorem, Convergence in Measure.

#### The Lebesgue Integral .

Simple and Step Functions, Lebesgue integral of step functions, Upper Functions, Lebesgue integral of upper functions, Lebesgue Integrable functions, Fatou's Lemma, Dominated Convergence Theorem, Monotone Convergence Theorem, Riemann integral as a Lebesgue integral, Lebesgue-Vitali Theorem, Application of the Lebesgue Integral.

#### References :

Aliprantis, C.D., Burkinshaw, O., *Principles of Real Analysis*, 3rd Edition, Harcourt Asia Pte Ltd., 1998.  
Royden, H.L., *Real Analysis*, 3rd Edition, Macmillan, New York & London, 1988.  
Halmos, P.R., *Measure Theory*, Van Nostrand, New York, 1950.  
Rudin, W., *Real and Complex Analysis*, McGraw-Hill Book Co., 1966.  
Kolmogorov, A.N., Fomin, S.V., *Measures, Lebesgue Integrals, and Hilbert Space*, Academic Press, New York & London, 1961.

**Note :** This course is based on book (1), Chapters 3, 4.

## Unit 1.3 : ( Complex Analysis)

### **Complex Numbers :**

Complex Plane, Lines and Half Planes in the complex plane, Extended plane and its Spherical Representation, Stereographic Projection.

### **Complex Differentiation :**

Derivative of a complex function, Comparison between differentiability in the real and complex senses, Cauchy-Riemann Equations, Necessary and Sufficient Criterion for complex differentiability, Analytic functions, Entire functions, Harmonic functions and Harmonic conjugates.

### **Complex Functions and Conformality :**

Polynomial functions, Rational functions, Power series, Exponential, Logarithmic, Trigonometric and Hyperbolic functions, Branch of a logarithm, Analytic functions as mappings, Conformal maps, Möbius Transformations.

### **Complex Integration :**

The complex integral (over piecewise  $C^1$  curves), Cauchy's Theorem and Integral Formula, Power series representation of analytic functions, Morera's Theorem, Goursat's Theorem, Liouville's Theorem, Fundamental Theorem of Algebra, Zeros of analytic functions, Identity Theorem, Weierstrass Convergence Theorem, Maximum Modulus Principle and its applications, Schwarz's Lemma, Index of a closed curve, Contour, Index of a contour, Simply connected domains, Cauchy's Theorem for simply connected domains.

### **Singularities :**

Definitions and Classification of singularities of complex functions, Isolated singularities, Laurent series, Casorati-Weierstrass Theorem, Poles, Residues, Residue Theorem and its applications to contour integrals, Meromorphic functions, Argument Principle, Rouché's Theorem.

### **Analytic Continuation :**

Schwarz Reflection Principle, Analytic Continuation along a path, Monodromy Theorem.

### **References :**

Conway, J.B., *Functions of one complex variable*, Second Edition, Narosa Publishing House.

Sarason, D., *Complex Function Theory*, Hindustan Book Agency, Delhi, 1994.

Ahlfors, L.V., *Complex Analysis*, McGraw-Hill, 1979.

Rudin, W., *Real and Complex Analysis*, McGraw-Hill Book Co., 1966.

Hille, E., *Analytic Function Theory* (2 vols.), Gonn & Co., 1959.

Titchmarsh, E.C., *The Theory of Functions*, Oxford University Press, London.

Ponnusamy, S., *Foundations of Complex Analysis*, Narosa Publishing House, 1997.

**Note :** This course is based on the books (1) and (2), as described below:

Section (i) : Books (1) & (2), Chapter I. Section (ii) : Book (2), Chapter II.

Section (iii) : Book (1), Chapter III. Section (iv) : Book (2), Chapters VI, VII, IX.

Section (v) : Book (1), Chapter V & Book (2), Chapter VIII. Section (vi) : Book (1), Chapter IX.

## Unit 1.4 : General Mechanics ( 50 Marks)

Generalized coordinates. Virtual work. D'Alemberts principle. Unilateral and bilateral constraints. Holonomic and Non-holonomic systems. Scleronomic and Rheonomic systems. Lagrange's equations of first and second kind. Uniqueness of solution. Energy equation for conservative fields. Euler's dynamical equations. Rotating coordinate system. Motion related to rotating earth. Foucault's pendulum and torque free motion of a rigid body about a fixed point. Motion of a symmetrical top and theory of small vibrations.

Hamilton's variables. Hamilton canonical equation. Homogeneity of space and time conservation principles, Noethers theorem. Cyclic coordinates. Routh's equations. Hamilton's principle. Principle of least action. Poisson's Bracket. Poisson's identity. Jacobi-Poisson Theorem.

Time dependent Hamilton-Jacobi equation and Jacobi's Theorem. Lagrange Brackets. Condition of canonical character of transformation in terms of Lagrange brackets and Poisson brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations.

### References :

1. H. Goldstein : Classical Mechanics.
2. N.C. Rana and P.S. Jog : Classical Mechanics.
3. Louis N. Hand and Janet D. Finch : Analytical Mechanics.
4. A.S. Ramsay : Dynamics Part – II.
5. S.L. Loney : Rigid Dynamics.

## Unit 1.5 : Differential Geometry ( 50 Marks)

### Tensors:

Tensor and their transformation laws, Tensor algebra, Contraction, Quotient law, Reciprocal tensors, Kronecker delta, Symmetric and skew-symmetric tensors, Metric tensor, Riemannian space, Christoffel symbols and their transformation laws, Covariant differentiation of a tensor, Riemannian curvature tensor and its properties, Bianchi identities, Ricci-tensor, Scalar curvature, Einstein space.

### Curves in Space:

Parametric representation of curves, Helix, Curvilinear coordinates in  $E_3$ . Tangent and first curvature vector, Frenet formulas for curves in space, Frenet formulas for curve in  $E_n$ . Intrinsic differentiation, Parallel vector fields, Geodesic.

### Surfaces :

Parametric representation of a surface, Tangent and Normal vector field on a surface, The first and second fundamental tensor, Geodesic curvature of a surface curve, The third fundamental form, Gaussian curvature, Isometry of surfaces, Developable surfaces, Weingarten formula, Equation of Gauss and Codazzi, Principal curvature, Normal curvature, Meusnier's theorem.

### References :

1. Tensor Calculus and Application to Geometry and Mechanics : (chapter-II and III) – I.S.SOKOLNIKOFF.
2. An Introduction to Differential Geometry: (chapter – I,II,III,V and VI) - T.T.WILMORE.
3. Differential Geometry:- BARY SPAIN.

## M.Sc. First Year Second Semester (250 marks)

### Unit 2.1 : Algebra II (50 Marks)

#### Fields (40 Marks) :

Field Extensions : Algebraic and Transcendental Extensions, Finite Extension, Algebraic Closure of a field, Algebraically Closed Field, Splitting Field of a polynomial, Normal Extension, Separable Extension, Impossibility of some constructions by straightedge and compass.

Finite Fields and their properties, Galois Group of automorphisms and Galois Theory, Solution of polynomial equations by radicals, Insolvability of the general equation of degree 5(or more) by radicals.

**Linear Algebra (10 Marks) :** Canonical Forms : Similarity of linear transformations, Diagonalization, Invariant Subspaces, Reduction to Triangular Forms.

Nilpotent Transformations, Index of Nilpotency, Invariants of a nilpotent transformation, Jordan Blocks and Jordan Forms, Rational Canonical Form, Generalized Jordan Form over an arbitrary field.

#### References :

Dummit, D.S., Foote, R.M., *Abstract Algebra*, Second Edition, John Wiley & Sons, Inc., 1999.

Goldhaber, J.K., Ehrlich, G., *Algebra*, The Macmillan Company, Collier-Macmillan Limited, London.

Herstein, I.N., *Topics in Abstract Algebra*, Wiley Eastern Limited.

Hungerford, T.W., *Algebra*, Springer.

Jacobson, N., *Basic Algebra, I & II*, Hindusthan Publishing Corporation, India.

Malik, D.S., Mordesen, J.M., Sen, M.K., *Fundamentals of Abstract Algebra*, The McGraw-Hill Companies, Inc.

Rotman, J.J., *The Theory of Groups: An Introduction*, Allyn and Bacon, Inc., Boston.

## Unit 2.2 : Topology (50 Marks)

### Set Theory :

Countable and Uncountable Sets, Schroeder-Bernstein Theorem, Cantor's Theorem, Cardinal Numbers and Cardinal Arithmetic, Continuum Hypothesis, Zorn's Lemma, Axiom of Choice, Well-Ordered Sets, Maximum Principle, Ordinal Numbers.

### Topological Spaces and Continuous Functions :

Topological spaces, Basis and Subbasis for a topology, Order Topology, Product topology on  $X \times Y$ , subspace Topology, Interior Points, Limit Points, Derived Set, Boundary of a set, Closed Sets, Closure and Interior of a set, Kuratowski closure operator and the generated topology, Continuous Functions, Open maps, Closed maps and Homeomorphisms, Product Topology, Quotient Topology, Metric Topology, Complete Metric Spaces, Baire Category Theorem.

### Connectedness and Compactness :

Connected and Path Connected Spaces, Connected Sets in  $\mathbb{R}$ , Components and Path Components, Local Connectedness.

Compact Spaces, Compact Sets in  $\mathbb{R}$ , Compactness in Metric Spaces, Totally Bounded Spaces, Ascoli-Arzelà Theorem, The Lebesgue Number Lemma, Local Compactness.

### References :

Munkres, J.R., *Topology, A First Course*, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

Dugundji, J., *Topology*, Allyn and Bacon, 1966.

Simmons, G.F., *Introduction to Topology and Modern Analysis*, McGraw-Hill, 1963.

Kelley, J.L., *General Topology*, Van Nostrand Reinhold Co., New York, 1955.

Hocking, J., Young, G., *Topology*, Addison-Wesley Reading, 1961.

Steen, L., Seebach, J., *Counter Examples in Topology*, Holt, Reinhart and Winston, New York, 1970.

**Note :** This course is based on the book (1), Chapters 1 - 5. 0.5in



## Unit 2.3 : Functional Analysis (50 Marks)

### Banach Spaces :

Normed Linear Spaces, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Quotient Space of normed linear spaces and its completeness, Riesz Lemma, Fixed Point Theorems and its applications.

Bounded Linear Transformations, Normed linear spaces of bounded linear transformations, Uniform Boundedness Theorem, Principle of Condensation of Singularities, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Hahn-Banach Theorem, Dual Space, Reflexivity of Banach Spaces.

### Hilbert Spaces :

Real Inner Product Spaces and its Complexification, Cauchy-Schwarz Inequality, Parallelogram law, Pythagorean Theorem, Bessel's Inequality, Gram-Schmidt Orthogonalization Process, Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval's Identity, Structure of Hilbert Spaces, Orthogonal Complement and Projection Theorem.

Riesz Representation Theorem, Adjoint of an Operator on a Hilbert Space, Reflexivity of Hilbert Spaces, Self-adjoint Operators, Positive Operators, Projection Operators, Normal Operators, Unitary Operators.

Introduction to Spectral Properties of Bounded Linear Operators.

### References :

Aliprantis, C.D., Burkinshaw, O., *Principles of Real Analysis*, 3rd Edition, Harcourt Asia Pte Ltd., 1998.

Goffman, C., Pedrick, G., *First Course in Functional Analysis*, Prentice Hall of India, New Delhi, 1987.

Bachman, G., Narici, L., *Functional Analysis*, Academic Press, 1966.

Taylor, A.E., *Introduction to Functional Analysis*, John Wiley and Sons, New York, 1958.

Simmons, G.F., *Introduction to Topology and Modern Analysis*, McGraw-Hill, 1963.

Limaye, B.V., *Functional Analysis*, Wiley Eastern Ltd.

Conway, J.B., *A Course in Functional Analysis*, Springer Verlag, New York, 1990.

Kreyszig, E., *Introductory Functional Analysis and its Applications*, John Wiley and Sons, New York, 1978.

**Note :** This course is based on book (1), Chapters 5, 6.

## Unit 2.4 : Mechanics of Continua(50 Marks)

Principles of continuum mechanics, axioms. Forces in a continuum. The idea of internal stress. Stress tensor. Equations of equilibrium. Symmetry of stress tensor. Stress transformation laws. Principal stresses and principal axes of stresses. Stress invariants. Stress quadric of Cauchy. Shearing stresses. Mohr's stress circles.

Deformation. Strain tensor. Finite strain components in rectangular Cartesian coordinates. Infinitesimal strain components. Geometrical interpretation of infinitesimal strain components. Principal strain and principal axes of strain. Strain invariants. The compatibility conditions. Compatibility of strain components in three dimensions.

Constitutive equations. Inviscid fluid. Circulation. Kelvin's energy theorem. Constitutive equation for elastic material and viscous fluid. Navier and Stokes equations of motion.

Motion of deformable bodies. Lagrangian and Eulerian approaches to the study of motion of continua. Material derivative of a volume integral. Equation of continuity. Equations of motion. Equation of angular momentum. Equation of Energy. Strain energy density function.

### References :

1. Y.C. Fung : A first course in continuum mechanics.
2. A.C. Eringen : Mechanics of continua.
3. L.I. Sedov : A course in continuum mechanics. Vol – I.
4. W. Prager : Mechanics of continuous media.

## Unit 2.5 : Ordinary Differential Equation and Special Function (50 Marks)

First order ODE, Initial value problems, existence theorem, basic theorems, Ascoli Arzela theorem, Theorem on convergence of solution of initial value problems, Picard - Lindelof theorem, Peano's existence theorem and corollaries. Variation of parameters, Lagrange's identity, adjoint and self-adjoint equations. Green's function.

Linear differential equation in complex plane, Fundamental system of integrals, singularities of differential equation, Integrals near a nonessential singularity, Regular integrals, Contour integral solution of Hypergeometric equation, Legendre equation, Bessel equation in the complex plane and their properties.

### References :

1. Coddington, E.A and Levinson, N., Theory of ordinary differential equation, McGraw Hill.
2. Estham, Ordinary differential equation.
3. Hartman, P, Ordinary differential equation, John Wiley and sons
4. Reid, W.T. Ordinary differential equation, John Wiley and sons.
5. Burkhill, J.C., Theory of ordinary differential equation
6. Ince, E.L. Ordinary differential equation, Dover

## **M.Sc. Second year First Semester (250 Marks)**

### **Unit 3.1: Numerical Analysis (Theory :30, Practical: 20)**

#### **Numerical Analysis Theory ( 30 Marks):**

##### **System of linear equations and eigenvalue problem :**

Operational counts for direct methods of solving system linear algebraic equations. Gaussian operational count for inversion of a matrix. Eigenvalue problem. General iterative method. Jacobi and Gauss. Seidel method. Relaxation method. Necessary and sufficient conditions for convergence. Speed of convergence. S.O.R. and S.U.R. methods. Gerschgorin's circle theorem. Determination of eigenvalue by iterative methods.. Ill conditioned system .

##### **System of non-linear method equations :**

Newton's method. Existence of roots. Stability and convergence under variation of initial approximations. General iterative method for the system :  $x = g(x)$  and its sufficient condition for convergence. The method of steepest descent.

##### **Finite difference method :**

Solution of partial differential equations by finite difference method. Partial difference quotients. Discretization error. Idea of convergence and stability. Explicit and Crank-Nicolson implicit method of solution of one dimensional heat conduction equation : convergence and stability. Standard and diagonal five point formula for solving Laplace and Poisson equations. Explicit and Implicit method of solving Cauchy problem of one-dimensional wave equation. CFL conditions of stability and convergence. Finite difference approximations in polar coordinates.

#### **Numerical Analysis Practical ( 20 Marks) :**

##### **List of Practical Problems**

1. Gauss-Jordon method.
2. Inverse of a matrix
3. S.O.R. / S.U.R. method
4. Relaxation method
5. Solution of one dimensional heat conduction equation by
  - i) Explicit and
  - ii) Crank-Nicolson implicit method.
6. Solution of Laplace equation.
7. Solution of Poisson equation.
8. Solution of one-dimensional wave equation.

## References :

1. Computing methods ; Berzin and Zhidnov.
2. Analysis of Numerical methods : Isacson and Keller.
3. A first course in Numerical Analysis : Ralston and Rabinowitz.
4. Numerical solution of differential equations : M.K.Jain.
5. Numerical solution of partial differential equations : G.D.Smith.
6. The finite element method in structural and continuum mechanics : O.C.Zienkiewics.
7. The finite elements method in partial differential equations : A.R.Mitchell.
8. An introduction to boundary element methods : Prem K. Kytbe.
9. Computational Mathematics : B.P.Demidovich and J.A.Maron.
10. Applied Numerical Methods : A. Gourdin & M. Boumahrat.

## Unit 3.1: Advanced Topology

### Countability and Separation Axioms :

Countability Axioms, The Separation Axioms, Equation spaces, Lindelöf spaces, Regular spaces, Normal spaces, Urysohn Lemma, Tietze Extension Theorem.

### Nets and Filters :

Directed Sets, Nets and Subnets, Convergence of a net, Ultranets, Partially Ordered Sets and Filters, Convergence of a filter, Ultrafilters, Basis and Subbase of a filter, Nets and Filters in Topology.

### Tychonoff Theorem & Compactification :

Tychonoff Theorem, Completely Regular spaces, Local Compactness, One-point compactification, Stone-Cech Compactification.

### Metrization:

Urysohn Metrization Theorem, Topological Imbedding, Imbedding Theorem of a regular space with countable base in  $\mathbb{R}^n$ , Partitions of Unity, Topological  $m$ -Manifolds, Imbedding Theorem of a compact  $m$ -manifold in  $\mathbb{R}^n$ .

Local Finiteness, Nagata-Smirnov Metrization Theorem, Paracompactness, Stone's Theorem, Local Metrizability, Smirnov Metrization Theorem.

Uniform Spaces.

### Complete Metric Spaces & Function Spaces:

Complete Metric Spaces, The Peano Space-Filling Curve, Hahn-Mazurkiewicz Theorem (statement only).

Compactness in Metric Spaces, Equicontinuity, Pointwise and Compact Convergence, The Compact-Open Topology, Stone-Weierstrass Theorem, Ascoli's Theorem, Baire Spaces, A Nowhere Differentiable Function.

An Introduction to Dimension Theory, Topological notion of (Lebesgue)dimension.

**References :**

Munkres, J.R., *Topology, A First Course*, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.  
Dugundji, J., *Topology*, Allyn and Bacon, 1966.  
Simmons, G.F., *Introduction to Topology and Modern Analysis*, McGraw-Hill, 1963.  
Kelley, J.L., *General Topology*, Van Nostrand Reinhold Co., New York, 1955.  
Bourbaki, N., *Topologie Générale*.  
Hocking, J., Young, G., *Topology*, Addison-Wesley Reading, 1961.  
Steen, L., Seebach, J., *Counter Examples in Topology*, Holt, Reinhart and Winston, New York, 1970.

**Note :** This course is based on the books [1]; Chapters 4 - 7 and [4]. 0.6in

**Unit 3.1: Discrete Mathematics – I****Graph Theory**

Introduction to Graphs : The concept of a graph, Paths in graphs, Graphs and graph models, Graph terminology and special types of graphs, Bipartite graphs, Complete graphs, External graphs, Intersection graphs, Operations on graph, Graph Isomorphism.

Blocks : Cutpoints, bridges and blocks. Block graphs and cutpoint graphs.

Trees : Introduction to trees and characterizations, Applications of Trees, Spanning Trees, Minimum Spanning Trees, Trees in computer science, Centers and centroids, Block-cutpoint trees, Independent cycles and cocycles, Matroids.

Connectivity : Connectivity and line-connectivity, Graphical version of Menger's theorem.

Traversability : Eulerian Graphs, Hamiltonian Graphs.

Coverings and Matching : Coverings and independence, Critical points and lines, Matching, Maximum Matching Problem, Minimum covering problems.

Representing Graphs : Adjacency matrix, Incidence matrix, Cycle matrix.

Planarity : Plane and planar graphs, Outerplanar graphs, Kuratowski's theorem, other characterizations of planar graphs.

Colorability : Vertex coloring, Chromatic number, Edge coloring, Five color theorem, Four color conjecture, Unique colorable graphs.

Directed Graphs : Basic definitions, Type of Connectedness, Covers and Bases, Distance concepts and matrices, Connectivity, Acyclic digraphs, Cycles and traversability, Orientations and Tournaments.

Network Flows : Max Flow – Min Cut Theorem, Menger's Theorem.

**Text Books :**

1. Graph Theory, F. Harary, Narosa Publishing House, 1993.

**References :**

1. Introduction to Graph Theory, Douglas B. West, Prentice-Hall of India Pvt. Ltd., New Delhi 1999.
2. Basic Graph Theory, K.R. Parthasarathi, Tata McGraw-Hill Publ. Co. Ltd., New Delhi, 1994.
3. Graph Theory with Applications to Engineering and Computer Science, Narsingh Deo, Prentice-Hall of India Pvt.Ltd., New Delhi, 1997.
4. Applied Combinatorics, Fred S. Roberts, Prentice-Hall of India Pvt. Ltd.

### **Unit 3.2: PDE and Nonlinear ODE**

**Theory of Partial Differential Equations**

Introduction, Cauchy-kowalewski's theorem (statement only) classification of second order partial differential equations to Hyperbolic, Elliptic and Parabolic types. Reduction of linear and quasilinear equations in two independent variables to their canonical forms, characteristic curves. Well-posed and ill-posed problems.

**(i) Hyperbolic Equations:**

The equation of vibration of a string. Formulation of mixed initial and boundary value problem. Existence, uniqueness and continuous dependence of the solution to the initial conditions. D'Alembert's formula for the vibration of an infinite string. The domain of dependence, the domain of influence use of the method of separation of variables for the solution of the problem of vibration of a string. Investigation of the conditions under which the infinite series solution converges and represents the solution. Riemann method of solution, Problems, Transverse vibration of membranes. Rectangular and circular membranes problems.

**(ii) Elliptic Equations :**

Occurrence of Laplace's equation. Fundamental solutions of Laplace's equation in two and three independent variables. Laplace equation in polar, Spherical polar and in cylindrical polar coordinates, Minimum – Maximum theorem and its consequences. Boundary value problems, Dirichlet and Neumann's interior and exterior problems uniqueness and continuous dependence of the solution on the boundary conditions. Use of the separation of variables method for the solution of Laplace's equations in two and three dimensions interior and exterior Dirichlet's problem for a circle, and a semi circle, steady-state heat flow equation Problems, Higher dimensional problems, Dirichlet's problem for a cube, cylinder and sphere, Green's function for the Laplace equation, in two and three dimensions.

**(iii) Parabolic Equation :**

Conduction of heat in a bounded strip, First boundary value problem Maximum-Minimum theorem and its consequences , uniqueness, continuous dependence of the solution and existence of the solution. Conduction of heat in a infinite strip (Cauchy problem), Problems.

**Non-linear ODE**

Flows , phase space , existence and uniqueness of solution (statement only) Defination of stability , Lyapunov function , fixed points and their nature , saddlepoint , node , focus points , stable , unstable and center subspaces . Hartman-Grotman theorem (statement only) , Poincare map , periodic orbits , invariant sets ,limit points and limit cycles , attracting and repelling sets , trapping regions , two dimensional flow , Poincare Benedixon theorem (statement only) bifurcation .

**References :**

1. Sneddon I.N. : Elements of Partial Differential Equations, Mcgraw Hill.
2. Williams W.E. : Partial Differential Equations.
3. Miller F.H. : Partial Differential Equations
4. Petrovsky. I.G : Lectures on Partial differential equations.
5. Courant and Hilbert : Methods of Mathematical Physics, Vol – II

### Unit 3.3 : Any One Course from A -1.1 to A-1.8\*

A - 1.1	:	Advanced Algebra – I
A - 1.2	:	Algebraic Topology
A - 1.3	:	Introduction to Algorithms - I
A - 1.4	:	Fluid Mechanics I
A - 1.5	:	Mathematical Modelling of Biological Systems I
A - 1.6	:	Mathematical Theory of Elasticity I
A - 1.7	:	Principles of Operations Research I
A - 1.8	:	Non-Relativistic Quantum Mechanics

(\* Choice of course in unit 4.3 will depend on this course, if a student opts for A-1.5 then the student has to go for A-2.5 in unit 4.3)

#### A – 1.1 : Advanced Algebra - I ( 50 Marks )

##### Modules Theory (20 Marks) :

Modules and Module Homomorphisms, Submodules and Quotient Modules, Operations on submodules, Direct Sum and Product, Finitely Generated Modules, Free Modules. Tensor Products of modules, Universal Property of the tensor product, Restriction and Extension of Scalars, Algebras.

Exact Sequences, Projective, Injective and Flat Modules, Five Lemma, Projective Modules and  $\text{Hom}_{\mathbb{R}}(M, -)$ , injective modules and  $\text{Hom}_{\mathbb{R}}(-, M)$ , Flat modules and  $M \otimes_{\mathbb{R}} -$ .

**Note :** This course is based on the books [2]; Chapter 2 and [1]; Chapter 10.

##### Commutative Ring Theory (30 Marks) :

Rings and Ring Homomorphisms, Ideals, Quotient Rings, Zero-divisors, Nilpotent elements, Units, Prime and Maximal ideals, Nilradical and Jacobson radical, Nakayama's Lemma, Operations on Ideals, Prime Avoidance, Chinese Remainder Theorem, Extension and Contraction of ideals.

Rings and Modules of Fractions, Local Properties, Extended and contracted ideals in rings of fractions.

Noetherian Rings, Primary Decomposition in Noetherian Rings.

Integral Dependence, Lying-Over Theorem, Going-Up Theorem, Integrally Closed Domains, Going-Down Theorem, Noether Normalization, Hilbert Nullstellensatz.

Transcendence Base, Separably Generated Extensions, Schmidt and Lüroth Theorems.

**Note :** This course is based on the book [2]; Chapters 1, 3, 4, 5.



## References :

- Dummit, D.S., Foote, R.M., *Abstract Algebra*, Second Edition, John Wiley & Sons, Inc., 1999.
- Atiyah, M., MacDonald, I.G., *Introduction to Commutative Algebra*, Addison-Wesley, 1969.
- Lang, S., *Algebra*, Addison-Wesley, 1993.
- Lam, T.Y., *A First Course in Non-Commutative Rings*, Springer Verlag.
- Hungerford, T.W., *Algebra*, Springer.
- Jacobson, N., *Basic Algebra, II*, Hindusthan Publishing Corporation, India.
- Malik, D.S., Mordesen, J.M., Sen, M.K., *Fundamentals of Abstract Algebra*, The McGraw-Hill Companies, Inc.
- Curtis, C.W., Reiner, I., *Representation Theory of Finite Groups and Associated Algebras*, Wiley-Interscience, NY.

## A – 1.2 : Algebraic Topology : 50 Marks

### Algebraic Topology (50 Marks)

#### The Fundamental Group and Covering Spaces (25 marks) :

Homotopy of paths, Fundamental Group, Covering Spaces, Fundamental Group of the Circle, Fundamental Group of the Punctured Plane, Special Van Kampen Theorem, Fundamental Group of  $S^n$ , Seifert - Van Kampen Theorem (statement and applications), Fundamental Group of Surfaces.

Essential and Inessential Maps, Borsuk - Ulam Theorem for  $S^2$ , Fundamental Theorem of Algebra, Vector Fields and Fixed Points, Brouwer's Fixed-Point Theorem for the disc, Homotopy Type, Deformation Retract, Strong Deformation Retract.

Jordan Separation Theorem, Jordan Curve Theorem (statement only).

Classification of Covering Spaces, General Lifting Lemma, Existence of Coverings, Semilocally Simply Connectivity, Deck Transformations.

**Note :** This course is based on the book [1]; Chapter 8.

#### Simplicial Homology (25 Marks) :

Geometric Complexes and Polyhedra, Orientation of Geometric Complexes.

Chains, Cycles, Boundaries and Homology Groups, Examples of Homology Groups, The Structure of Homology Groups, Euler - Poincaré Theorem, Pseudomanifolds and the Homology Groups of  $S^n$ .

Simplicial Approximation, Induced Homomorphisms on the Homology Groups, Brouwer Fixed Point Theorem and Related Results.

**Note :** This course is based on the book [2]; Chapters 1 - 3.

## References :

- Munkres, J.R., *Topology, A First Course*, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
- Croom, F.H., *Basic Concepts of Algebraic Topology*, Springer, NY, 1978.
- Bredon, G.E., *Topology and Geometry*, Springer, India, 2005.
- Spanier, E.H., *Algebraic Topology*, McGraw-Hill, 1966.
- Singer, I.M., Thorpe, J.A., *Lecture Notes on Elementary Topology and Geometry*, Springer, India, 2003 .
- Hatcher, A., *Algebraic Topology*, Cambridge University Press, 2003.
- Dieudonné, J., *A History of Algebraic and Differential Topology, 1900 - 1960*, Birkhäuser, 1989.

## **A – 1.3 : Introduction to Algorithms - I : 50 marks**

### **Theory - 35, Assignment - 15 (Computer lab access is necessary and mandatory)**

Algorithms : Introduction and Basic Concepts: Introduction to complexity of algorithms and different asymptotic notations. Complexity measures, worst-case and average-case complexity functions, problem complexity. 10 Lectures

Induction and Recursion : Mathematical Induction, Strong Induction and Well-Ordering, Recursive Definitions and Structural Induction, Recursive Algorithms, Program Correctness 10 Lectures

Designing Algorithms: Recurrence relations, Solving linear recurrence relations, Divide-and-conquer approach, The Master method. 10 Lectures

Data Structure: Introduction to ADT, implementations of basic data structures, array, stack, queue, dequeue, priority queue, linked list, binary tree and traversal algorithms, threaded tree, m-ary tree, heap, generalized list and garbage collection. 15 Lectures

Searching: Fibonacci search, binary search tree, Searching in static table - binary search, path lengths in binary trees and applications, optimality of binary search in worst case and average-case, binary search trees, construction of optimal weighted binary search trees; Searching in dynamic table - randomly grown binary search trees, AVL and (a,b) trees. 20 Lectures

Hashing: Techniques, analysis with chaining and open addressing. 7 Lectures

Sorting and selection: Finding maximum and minimum, k largest elements in order; Sorting by selection, tournament and heap sort, lower bound for sorting, quick sort, merge sort and sorting in linear time; Selection of k-th largest element. 20 Lectures

Union-Find problem: Tree representation of a set, weighted union and path compression-analysis and applications. 8 lectures

**Text books :**

1. T. H. Cormen, C.E. Leiserson and R.L.Rivest: Introduction to Algorithms, Prentice Hall of India, New Delhi, 1998.

**References:**

1. A. M. Tannenbaum and M. J. Augesestein: Data Structures Using PASCAL, Prentice Hall, New Jersey, 1981.
2. E. Horowitz and S. Sahni: Fundamentals of Data Structures, CBS, New Delhi, 1977.
3. A. Aho, J. Hopcroft, and J. Ullman: Data Structures and Algorithms, Addison-Wesley, Reading, Mass., 1983.
4. E. Horowitz and S. Sahni: Fundamental of Computer Algorithms, Galgotia Pub./Pitman, New Delhi/London, 1987/1978.

**A– 1.4 : Fluid Mechanics I (50 marks)**

Lagrange's and Euler's methods in fluid motion. Equation of motion and equation of continuity, Boundary conditions and boundary surface stream lines and paths of particles. Irrotational and rotational flows, velocity potential. Bernoulli's equation. Impulsive action equations of motion and equation of continuity in orthogonal curvilinear coordinate. Euler's momentum theorem and D'Alemberts paradox.

Theory of irrotational motion flow and circulation. Permanence irrotational motion. Connectivity of regions of space. Cyclic constant and acyclic and cyclic motion. Kinetic energy. Kelvin's minimum. Energy theorem. Uniqueness theorem.

Dimensional irrotational motion.

Function. Complex potential, sources, sinks, doublets and their images circle theorem.

Theorem of Blasius. Motion of circular and elliptic cylinders. Circulation about circular and elliptic cylinder. Steady streaming with circulation. Rotation of elliptic cylinder.

Theorem of Kutta and Juokowski. Conformal transformation. Juokowski transformation. Schwartz-chirstoffel theorem.

Motion of a sphere. Stoke's stream function. Source, sinks, doublets and their images with regards to a plane and sphere.

Vortex motion. Vortex line and filament equation of surface formed by stream lines and vortex lines in case of steady motion. Strength of a filament. Velocity field and kinetic energy of a vortex system. Uniqueness theorem rectilinear vortices. Vortex pair. Vortex doublet. Images of a vortex with regards to plane and a circular cylinder. Angle infinite row of vortices. Karman's vortex sheet

Waves: Surface waves. Paths of particles. Energy of waves. Group velocity. Energy of a long wave.

## Reference Books:

1. Hydrodynamics –A.S.Ramsay(Bell)
2. Hydrodynamics – H. Lamb(Cambridge)
3. Fluid mechanics – L.D.Landou and E.M.Lifchiz(Pergamon),1959
4. Theoretical hydrodynamics –L.M.Thomson
5. Theoretical aerodynamics –I.M.Milne-Thomson;Macmillan, 1958
6. Introduction to the theory of compressible flow –Shih-I.Pai; Van Nostrand, 1959
7. Inviscid gas dynamics – P.Niyogi, Mcmillan, 1975(india)
8. Gas dynamics – K.Oswatitsch(english tr.) academic press, 1956

## A – 1.5 : Mathematical Modelling of Biological Systems I : 50 Marks

### (Qualitative Theory of Linear and Nonlinear systems)

#### **Linear systems:**

Linear autonomous systems, existence, uniqueness and continuity of solutions, diagonalization of linear systems, fundamental theorem of linear systems, the phase paths of linear autonomous plane systems, complex eigen values, multiple eigen values, similarity of matrices and Jordon canonical form, stability theorem, reduction of higher order ODE systems to first order ODE systems, linear systems with periodic coefficients.

#### **Nonlinear systems:**

The flow defined by a differential equation, linearization of dynamical systems (two, three and higher dimension), Stability: (i) asymptotic stability (Hartman's theorem), (ii) global stability (Liapunov's second method).

#### **Periodic Solutions (Plane autonomous systems):**

Translation property, limit set, attractors, periodic orbits, limit cycles and separatrix, Bendixon criterion, Dulac criterion, Poincare-Bendixon Theorem, index of a point, index at infinity.

#### **Bifurcation and Center Manifolds:**

Stability and bifurcation, saddle-node, transcritical and pitchfork bifurcations, hopf-bifurcation, center manifold (linear approximation).

#### **Linear difference equations:**

Difference equations, existence and uniqueness of solutions, linear difference equations with constant coefficients, systems of linear difference equations, qualitative behavior of solutions to linear difference equations.

#### **Nonlinear difference equations (Map):**

Steady states and their stability, the logistic difference equation, systems of nonlinear difference equations, stability criteria for second order equations, stability criteria for higher order system.

#### **Chaos:**

One-dimensional logistic map and chaos.**References:**

1. D. W. Jordan and P. Smith (1998): Nonlinear Ordinary Equations- An Introduction to Dynamical Systems (Third Edition), Oxford Univ. Press.
2. L. Perko (1991): Differential Equations and Dynamical Systems, Springer Verlag.
3. F. Verhulst (1996): Nonlinear Differential Equations and Dynamical Systems, Springer Verlag.
4. Alligood, Sauer, Yorke (1997): Chaos- An Introduction to Dynamical Systems, Springer Verlag.
5. W. G. Kelley and A. C. Peterson (1991): Difference Equations- An Introduction with Applications, Academic Press.

### **A – 1.6 : Mathematical Theory of Elasticity I (50 Marks)**

**Analysis of Stress :** State of stress in a body. Stress equations of equilibrium. Symmetry of stress tensor. Boundary conditions in terms of given surface forces. Principal stresses and invariants. Cauchy stress. Quadratic surface. Maximum normal and and shear stresses, Mohr's diagram. Beltrami-Michel Compatibility equations. Problems.

**Analysis of Strains :** Infinitesimal strains. Strain quadratic of Cauchy. Principal strains and invariants. Saint Venants compatibility equations. Finite deformations. Problems.

**Equations of Elasticity :** Hooke's law. Generalized Hooke's law. Various cases of Elastic symmetry of a body. The strain energy function and its connection with Hooke's law. Betti's identity. Clapeyrons formula and Clapeyrons theorem. Fundamental boundary value problems. Uniqueness and existence of solutions. Saint Venant's principle.

**Inverse and semi-inverse methods of solution :** Extension, Bending, Torsion and Flexure of beams : Solution of torsion problem as Dirichlet or Neumann boundary value problem. Prandtl's Analogy. Conformal mapping and the general problem of Flexure. Transverse bending. Problem of Torsion and Flexure for circular and elliptic bar. Torsion of circular shafts of variable diameter.

**Plane problems :** Plane strain and plane stress. Generalized plane stress. Airy's stress function. Solution of plane problems by means of polynomials. General Equations of the plane problems in polar co ordinates.

Reference:

1. A Treatise on The Mathematical Theory of Elasticity – A. E. Love
2. Mathematical Theory of Elasticity - I. S. Sokolnikoff
3. Theory of Elasticity – S. Timoshenko and J. N. Goodier
4. Elasticity Theory and Applications – A. S. Saada
5. Foundations of Solid Mechanics – Y. C. Fung
6. Theory of Elasticity – Y. A. Amenzade
7. Applied Elasticity – Zhilun Xu
8. Wave Propagations in Elastic Solids – J. D. Achenbach
9. Elasto-dynamics – A. C. Eringen
10. Wave Motion in Elastic Solids – K. F. Graff
11. Applied Elastity – Chi-The Wang.

## A- 1.7 : Principles of Operations Research I : 50 marks

### 1. Allocation Problems

#### (i) *Transportation Problems*

Mathematical representation of transportation problems, Unbalanced transportation problems, Degenerate transportation problems, Resolution of degeneracy.

#### (ii) *Assignment Problems*

Mathematical representation of assignment problems, Reduction theorems, Solution methods of assignment problems, Hungarian method of zero assignment technique, Restriction assignment, Negative cost etc, Variations of assignment problem, Multiple optimal solution, Maximization in assignment problem, Unbalanced assignment problem

#### (iii) *Travelling Salesman Problem/ Routing Problem*

Origin of travelling salesman problem, Symmetrical and asymmetrical problems, Mathematical representation of problems, Solution techniques for such problems using zero assignment/unit assignment etc.

### 2. Competitive Strategy (Game Theoretic Problems)

Introduction, Minimax/Maximin criterion, Payoffs, Rectangular games, Strategies, Pure and Mixed strategy problems, Saddle point, Graphical method of solving  $2 \times n$  and  $m \times 2$  games, Dominance principle, Equivalence of rectangular games and L.P.P. solution by Simplex method.

### 3. Queueing Theory (Theory of Waiting Lines)

Introduction, Queueing system, Queue disciplines FIFO, FIFS, LIFO, SIRO, FILO etc. The Poisson process (Pure birth process), Arrival distribution theorem, Properties of Poisson process, Distribution of inter arrival times (exponential process), Markovian property of inter arrival times, Pure death process (Distribution of departures), Derivation of service time distribution, Analogy of exponential service times with Poisson arrivals, Erlang service time distribution, Kendals notations, Probabilistic queueing models  $(M=M=1) : (I=FCFS)$ , General Erlang model,  $(M=M=1) : (N=FCFS)$ ;  $(M=M=S) : (I=FCFS)$  and their properties.

#### Reference Books

1. Operations Research - S.D. Sharma
2. Operations Research - Kanti Swarup, P.K. Gupta and Manmohan
3. OR methods and Problems - Sasieni Maurice, Arther Yaspan, Lawrence Friedman
4. Operations Research - H.S. Taha
5. Operations Resarch - T.L. Satty

## A-1.8 Non-Relativistic Quantum Mechanics (50 Marks)

Bohr's postulates of Quantum Mechanics. De Broglie's Wave. Heisenberg's principle of uncertainty. Probabilistic description. Schrödinger equation: Square well potential. One dimensional harmonic oscillator. Minimum uncertainty product. Momentum eigen functions. Box normalization. Orbital angular momentum. Hydrogen atom. Different approaches to quantum mechanics: Schrödinger representation, Heisenberg approach. Harmonic oscillator and Angular momentum as examples.

Time independent perturbation theory. Spin: two component wave function. Pauli's spin matrices. Variation method. Ground state energy level of helium atom as an application of variational method. Indistinguishable particles. Exclusion principle. Multiplet structure of spectral line: exchange degeneracy. Zeeman effect- Normal and Anomalous. Pauli's equation. Density matrix. Application of density matrix for Fermions. Time dependent perturbation theory: Fermi's Golden Rule. Molecules: Hydrogen molecule: Heitler- London theory of homopolar bonding. SU(2), SU(3) matrices and their properties.

### References:

1. L.I.Schiff: "Quantum Mechanics" (Mc Graw Hill)
2. E. Merzbacher: "Quantum Mechanics" (John Wiley)
3. Y.R.Waghmare: "Fundamentals of Quantum Mechanics" (Wheeler Publications)
4. "Fundamentals of Quantum Mechanics" – J.E.House (Academic Publishers)
5. R.H.Landau: "Quantum Mechanics II" (John Wiley)
6. L.H.Ballentine "Quantum Mechanics" (World Scientific)
7. P.J.E. Peebles "Quantum Mechanics" (Prentice Hall)

## **Unit 3.4 and Unit 3.5 : Two courses from B-1.1 to B-1.39**

- B - 1.1 Advanced Complex Analysis I
- B - 1.2 Advanced Differential Geometry I
- B - 1.3 Artificial Intelligence & Soft Computing I
- B - 1.4 Astrophysics I
- B - 1.5 Combinatorial Mathematics I
- B - 1.6 Commutative Algebra and Algebraic Geometry I
- B - 1.7. Computational Biology I
- B - 1.8 Computational Fluid Dynamics I
- B - 1.9. Computational Solid Mechanics I
- B – 1.10 Computer Graphics
- B – 1.11 Database Management Systems
- B - 1.12 Coupled Fields of Solid Mechanics & Plasticity I
- B - 1.13 Differential Geometry and its application I
- B - 1.14 Dynamical Meteorology & Numerical weather prediction I
- B - 1.15 Dynamical Oceanography I
- B - 1.16. Elastodynamics I
- B - 1.17 General Theory of Relativity and Cosmology I
- B - 1.18 Generalized function & Wavelet Theory I
- B - 1.19 Graph Theory I
- B - 1.20 Information Theory & Coding I
- B - 1.21 Magneto Fluid Mechanics I
- B - 1.22 Mathematical Ecology I
- B - 1.23 Mathematical Statistics I
- B - 1.24 Measure and Topology I
- B - 1.25 Mechanics of Viscous Fluid and Boundary Layer Theory I
- B - 1.26 Nonlinear and Dynamic Programming I
- B - 1.27 Number Theory
- B - 1.28 Operating System
- B - 1.29 Operator Algebra I
- B - 1.30 Operator Theory I
- B - 1.31 Plasma Mechanics I
- B - 1.32 Probability & Stochastic Processes I
- B - 1.33 Production and Inventory Control I
- B - 1.34 Quantum Field Theory & Statistical Mechanics I
- B - 1.35. Queuing Theory and Game Theory I
- B - 1.36 Renewable Bio-economic Modelling and Epidemiology I
- B - 1.37 Theory of Marketing Decisions I
- B - 1.38. Theory of Semi-groups I
- B - 1.39 Topological Groups & Harmonic Analysis I

(\* Choice of course in unit 4.4 and 4.5 will depend on this course, if a student opts for B-1.19 and B-1.30 then the student has to go for B-2.19 and B-2.30 in unit 4.4 and 4.5 )



## **B – 1.1: Advanced Complex Analysis I (50 Marks)**

The Functions  $M(r)$ ,  $A(r)$ , Hadamard Theorem on Growth of  $\log M(r)$ , Schwarz Inequality, Borel-Caratheodory Inequality.

Entire Functions, Growth of an Entire Function, Order and Type and their Representations in terms of the Taylor Coefficients, Distribution of Zeros, Schottky's Theorem (no proof), Picard's Little Theorem, Weierstrass Factor Theorem, The Exponent of Convergence of Zeros, Hadamard Factorization Theorem, Canonical Product, Borel's First Theorem, Borel's Second Theorem (statement only).

Analytic Continuation, Natural Boundary, Analytic Element, Global Analytic Function, Concept of Analytic Manifolds, Multiple Valued Conditions, Branch Points and Branch Cut, Riemann Surfaces.

### **References :**

Conway, J.B., *Functions of one complex variable*, Second Edition, Narosa Publishing House.

Ahlfors, L.V., *Complex Analysis*, McGraw-Hill, 1979.

Rudin, W., *Real and Complex Analysis*, McGraw-Hill Book Co., 1966.

Hille, E., *Analytic Function Theory* (2 vols.), Gonn & Co., 1959.

Titchmarsh, E.C., *The Theory of Functions*, Oxford University Press, London.

Markusevich, A.I., *Theory of Functions of a Complex Variable*, Vol. I, II, III.

Copson, E.T., *An Introduction to the Theory of Functions of a Complex Variable*.

Hayman, W.K., *Meromorphic Functions*.

Kaplan, W., *An Introduction to Analytic Functions*.

## **B – 1.2 : Advanced Differential Geometry I (50 Marks)**

Topological Manifold, differentiable Manifold, Differentiable functions, Tangent and cotangent spaces, vector fields, Maps, Integral curves, Lie brackets, Lie algebra of vector fields, One parameter group of transformations, 1-forms, Submanifolds, Distribution, Tensor field.

Symmetrisation and alteration of fully contravariant and fully covariant tensors, Exterior algebra, Exterior derivative, Differential forms.

Lie groups and Lie algebras of Lie group, One parameter subgroups and Exponential maps, Homomorphisms and Isomorphisms, Lie transformation groups, General linear groups.

## References

1. Foundation of differential Geometry (vol-1) :- S.KOBAYASHI and K.NOMIZU.
2. An Introduction to Differentiable Manifolds and Riemannian Geometry :- W.M.BOOTHBY.
3. Introduction to Differentiable Manifolds :- L.AUSLANDER and R.E.MACKENZIE.
4. Lectures on Differential Geometry :- S.S.CHERN, W.H.CHEN and K.S.LAM.

## **B – 1.3 : Artificial Intelligence and Soft Computing I : 50 marks**

Introduction to Artificial Intelligence. Perspective of cognition. Production system. Search strategies for AI production system. Principles of logic programming. Knowledge representation. Structured approach to Knowledge representation. Rule based deduction system.

Introduction to Soft Computing. Components of Soft Computing – Fuzzy Logic, Artificial Neural networks, genetic algorithms.

### References:

1. Principles of Artificial Intelligence by Nils J. Nilsson – Springer.
2. Artificial Intelligence and Soft Computing by Amit Konar – CRC Press.
3. Neural Networks by Simon Haykin – Pearson
4. Genetic Algorithms by David E. Goldberg – Addison-Wesley

## **B – 1.4 : Astrophysics I (50 marks)**

### Application of General Relativity to Astrophysics

1. Compact Objects, White dwarfs, Neutron stars and Black holes. Brief history of the formation and evolution of stars.
2. Schwarzschild exterior solution, Birkhoff's theorem, Schwarzschild singularity, Kruskal transformation, Schwarzschild Black hole. Motion of test particles around Schwarzschild black hole. Kerr metric and Kerr black holes ( without deduction of solution ). Horizons of Schwarzschild and Kerr black holes. Laws of black hole thermodynamics ( statements only ).
3. Interior of Schwarzschild metric, massive objects, Oppenheimer – Volkoff limit, Gravitational lensing , Quasars , Pulsars, Supernova.
4. Oppenheimer-Snyder non static dust model, Gravitational collapse.
5. Accretion into compact objects, Boltzmann formula, Saha Ionization equation, H-R diagram.

### References:

1. The Structure of the Universe – J.V. narlikar
2. Astrophysics – B. Basu
3. Astrophysics – B. Basu
4. Astrophysical Concept – M. Harmitt
5. Galactic Structure – A. Blaauw & M. Schmidt
6. Large Scale Structure of Galaxies – W.B. Burton
7. The Milky Way – B.T. Bok & P.F. Bok.
8. Cosmic Electrodynamics – J.H. Piddington

### **B – 1.5 : Combinatorial Mathematics I (50 Marks)**

General Principles of Enumeration, Counting of Sub-Sets, Partitions, Binomial Theorem, Multinomial Theorem.

Principles of Inclusion and Exclusion, Derangements, Rook polynomials, Arrangement with Forbidden Positions.

General Principles of Enumeration, Counting.

Latin Square, Quasi-group, Orthogonal Latin Square.

### References :

1. COMBINATORICS – Topics, Techniques, Algorithms *by* Peter J. Cameron (Cambridge University Press).
2. A COURSE IN COMBINATORICS *by* J. H. Lint & R. M. Wilson (Cambridge University Press).
3. DISCRETE AND COMBINATORIAL MATHEMATICS *by* Ralph P. Grimaldi (AWL).
4. DISCRETE MATHEMATICS FOR COMPUTER SCIENTISTS & MATHEMATICIANS *by* Joe L. Mott, Abraham Kandel & Theodore P. Baker (Prentice-Hall).

## **B – 1.6: Commutative Algebra and Algebraic Geometry I (50 Marks)**

Commutative Rings and Ideals, Ring Homomorphisms, Zero-divisors, Nilpotent elements, Nilradical and Jacobson radical, Nakayama's Lemma, Prime Avoidance, Chinese Remainder Theorem.

Modules and Module Homomorphisms, Tensor Products of modules, Exact Sequences, Projective, Injective and Flat Modules, Five Lemma, Projective Modules and  $\text{Hom}_R(M, -)$ , injective modules and  $\text{Hom}_R(-, M)$ , Flat modules and  $M \otimes_R -$ .

Local rings, Localisation, Applications.

Noetherian Modules, Primary Decomposition, Associated Primes, Artinian Modules, Length of a Module.

Integral Dependence, Lying-Over Theorem, Going-Up Theorem, Integrally Closed Domains, Going-Down Theorem, Noether Normalization.

Transcendence Base, Separably Generated Extensions, Schmidt and Lüroth Theorems.

### **References :**

Atiyah, M., MacDonald, I.G., *Introduction to Commutative Algebra*, Addison-Wesley, 1969.

Gopalakrishnan, N.S., *Commutative Algebra*, Oxonian Press Pvt. Ltd., New Delhi, 1988.

Fulton, W., *Algebraic Curves*, W.A. Benjamin, INC., 1969.

Musili, C.S., *Algebraic Geometry for Beginners*, TRIM 20, Hindustan Book Agency, New Delhi, 2001.

Walker, R., *Algebraic Curves*, Dover, NY, 1962.

Dummit, D.S., Foote, R.M., *Abstract Algebra*, Second Edition, John Wiley & Sons, Inc., 1999.

Lang, S., *Algebra*, Addison-Wesley, 1993.

## **B – 1.7 : Computational Biology I (50 marks)**

### **1. Dynamic Modelling with Difference Equations.**

The Malthusian Model. Non-linear Models. Analyzing Non-linear Models. Variations on the Logistic Model. Discrete and Continuous Models.

### **2. Linear Models of Structured Populations.**

Linear Models and Matrix Algebra. Projection Matrices for Structured Models.

Eigenvectors and Eigenvalues. Computation of Eigenvectors and Eigenvalues.

### **3. Non-linear models of Interactions.**

A simple Predator-Prey Model. Equilibria of Multi-population Models. Linearization and Stability. Positive and Negative Interactions.

### **4. Modelling Molecular Evolution.**

Background on DNA. An Introduction to Probability. Conditional Probability. Matrix Models of Base Substitution. Phylogenetic Distances.

### **References :**

1. Elizabeth A. Allman and John A. Rhodes: *Mathematical Models in Biology: An Introduction*. Cambridge University Press (2004)

## **B –1. 8 : Computational Fluid Dynamics I (50 marks)**

### **Finite difference method:**

Classification of 2<sup>nd</sup> order partial differential equations - parabolic, hyperbolic and elliptic types. Governing equations of fluid dynamics, Introduction to finite difference discretization. Explicit and Implicit schemes. Truncation error, consistency, convergence and stability analysis. Thomas algorithm. ADI method for 2-D heat conduction problem. Splitting and approximate factorization for 2-D Laplace equation. Multigrid method. Upwind scheme, CFL stability condition. Lax-Wendroff and MacCormack schemes.

### **Finite Volume method:**

Preliminary concepts. Flux computation across quadrilateral cells. Reduction of a BVP to algebraic equations. Illustrative example like, solution of Dirichlet problem for 2-D Laplace equation. Conservation principles of fluid dynamics. Basic equations of viscous and inviscid flow. Basic equations in conservative form. Associated typical boundary conditions for Euler and Navier-Stokes equations. Grid generation using elliptic partial differential equations. Incompressible viscous flow field computation: Stream function vorticity formulation, Staggered grid, MAC method, SIMPLE algorithm.

### **References:**

1. P. Niyogi, S. K. Chakraborty and M. K. Laha- Introduction to computational fluid Dynamics, Pearson Education, Delhi 2005.
2. C. A. J. Fletcher- Computational Techniques for Fluid Dynamics, Vol-I and Vol-II, Springer, 1988.
3. R. Peyret and T. D. Taylor- Computational Methods for Fluid Flow, Springer 1983.
4. J. F. Thompson, Z.U.A Warsi and C. W. Martin, Numerical Grid Generation, Foundations and Applications, North Hollamm, 1985.
5. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Trans., Pergamon Press, 1989.
6. H. Schlichting and K. Gersten- Boundary Layer Theory, 8<sup>th</sup> Ed., Springer 2000.
7. J.D. Andersson , Computational Fluid Dynamics

## **B – 1.9 : Computational Solid Mechanics – I (Theory) – 50.**

Introduction to computational methods in continuum mechanics. Stages of developments and features of computational mechanics. Different methods.

Finite Element Method :

Basic concept of Finite Element Method. Discretization of the domain. Derivation of element equations. Connectivity of elements. Imposition of boundary conditions. Solution of equations. Examples. Finite Element error analysis. Convergence of solution. Two dimensional problems: Finite element formulation of some axisymmetric problems – Torsion of a cylindrical bars, Transverse deflection of membranes. Application of Finite element method to some plane elasticity problems.

Boundary Element Method :

Fundamental Concepts. Weighted Residual Techniques : Weak variational formulation, Galerkin method, Rayleigh-Ritz method. Choice of test functions. Boundary and Domain solutions. Basic integral equation. Fundamental solution. Boundary Integral Equation. Boundary elements. Potential problems using constant and linear elements. Bending of an elastic beam. Quadratic element. Examples. Two-dimensional elastostatic problems: Analytical formulation, Numerical implementation, Treatment of Edges and Corners, examples. Coupling of Finite Element Method and Boundary Element Method. Examples.

**References :**

1. Finite Element Method : J.N.Reddy.
2. Finite Element Procedures : K.J.Bathe
3. Introduction to Finite and Boundary Element Method : G.Beer and J.O.Watson.
4. The Finite Element Method – Vol.I (The Basis): O.C.Zienkiewicz and R.L.Taylor
5. The Finite Element Method – Vol. II (Solid Mechanics) : O.C.Zienkiewicz and R.L.Taylor.
6. Programming the Finite Element Method : I.M. Smith, D.V.Griffiths.
7. Boundary Elements- An Introductory Course : C.A.Brebbia and J.Dominguez.
8. An Introduction to Boundary Element Methods : P.K. Kathy.
9. The Boundary Element Method in Engineering ; A.A.Becker.
10. Computational Elasticity : M. Ameen.
11. Boundary Element Techniques : C.A.Brebbia, J.C.F.Tellers, L.C.Wrobel
12. Underlying Principles of Boundary Element Method : D.J.Cartwright.
13. Programming the Boundary Element Method – An introduction for Engineers : G. Beer.

## **B -1. 10. Computer Graphics (50 Marks)**

**Theory - 30, Assignment - 20** (Computer lab access is necessary and mandatory)

Introduction: Objective, applications, GKS/PHIGS, normalized co-ordinate system, aspect ratio.

Graphics system: Vector and raster graphics, various graphics display devices, graphics interactive devices, segmented graphics, attribute table.

Raster scan Graphics: Line drawing algorithms, circle/ellipse drawing algorithms, polygon filling algorithms.

Geometric transformation: Homogeneous co-ordinate system, 2D and 3D transformations, projection orthographic and perspective.

Curve and Surfaces: Curve approximation and interpolation, Lagrange, Hermite, Bezier and B-Spline curves/surfaces and their properties, curves and surface drawing algorithms.

Geometric modeling: 3D object representation and its criteria, edge/vertex list, constructive solid geometry, wire-frame model, generalized cylinder, finite element methods.

Clipping: Window and viewport, 2D and 3D clipping algorithms.

Hidden line and hidden surfaces: Concept of object- and image-space methods, lines and surface removal algorithms.

Intensify and color models: RGB, YIQ, HLS and HSV models and their conversions, gamma correction, half toning.

Rendering: Illumination models, polygon mesh, shading, transparency, shadow, texture.

Some advance topics/applications: (i) Animation and morphing, (ii) Virtual reality, (iii) User interface design, (iv) Fractal graphics, (v) Multimedia authoring, (vi) 3D visualization..

### **References:**

1. W. K. Giloi: Interactive Computer Graphics: Data Structure, Algorithms, Languages, Prentice Hall, Englewood Cliffs, 1978.
2. W. M. Newman and R. F. Sproull: Principles of Interactive Computer Graphics, McGraw-Hill, New Delhi, 1979.
3. D. Hearn and P. M. Baker: Computer Graphics, 2nd ed. Prentice Hall of India, New Delhi, 1997.
4. F. S. Hill: Computer Graphics, McMillan, New York, 1990.
5. D. P. Mukherjee: Fundamentals of Computer Graphics and Multimedia, Prentice Hall of India, New Delhi, 1999.

## **B – 1.11 Database Management Systems (50 Marks)**

**Theory - 30, Assignment - 20**      (*Computer lab access is necessary and mandatory*)

Introduction: Purpose of database systems, data abstraction and modeling, instances and schemes, database manager, database users and their interactions, data definition and manipulation language, data dictionary, overall system structure.      16 lectures

Entity-relationship model: Entities and entity sets, relationships and relationship sets, mapping constraints, E-R diagram, primary keys, strong and weak entities, reducing E-R diagrams to tables, trees or graphs, generalization and specialization, aggregation.      10 lectures

Relational model: Structure of a relational database, operation on relations, relational algebra, tuple and domain relational calculus, salient feature of a query language.      6 lectures

Structured query language: Description an actual RDBMS and SQL.      6 lectures

Normalization: Pitfalls in RDBMS, importance of normalization, functional, multi-valued and join dependencies, 1NF to 5NF, limitations of RDBMS.      10 lectures

Database tuning: Index selection and clustering, tuning of conceptual schema, de-normalization, tuning queries and views.      6 lectures

Query optimization: Importance of query processing, equivalence of queries, cost Estimation for processing a query, general strategies, bi-relational and multi-relational join algorithms, algebraic manipulation.      10 lectures

Crash recovery: Failure classification, transactions, log maintenance, check point implementation, shadow paging, example of an actual implementation.      8 lectures

Object oriented model: Nested relations, modeling nested relations as object model, extension of SQL, object definition and query language (ODL, OQL), object relational database model, storage and access methods. Active databases, Advanced trigger structures, SQL extensions.      8 lectures

### **References:**

1. H. F. Korth and A. Silberschatz: Database System Concepts, McGraw Hill, New Delhi, 1997.
2. R. A. Elmasri and S. B. Navathe: Fundamentals of Database Systems, 3rd ed., Addison-Wesley, 1998.



## **B – 1.12 : Coupled fields of Solid Mechanics and Plasticity I (50 marks)**

### **Theory of Plasticity :**

Octahedral stress. Spherical stress . Deviatoric stress. Invariants of stress. Mohr's stress circle. Equivalent stress. Invariants of strain. Strain-rate-its invariants. Partition of strain energy into spherical and deviatoric stress.

General form of yield condition for an ideal plastic. Tresca and von mises yield conditions- physical aspects.

Strain hardening and work hardening . Brucker's postulates, General form of flow equations for strain hardening materials. Non-hardening materials materials . Uniqueness theorem for non-hardening materials. Yield condition as consequence of plasticity conditions.

Elastoplastic problems. Bending and torsion of beams. Plane plastic problems.

### References

1. R. Hill : Mathematical theory of plasticity
2. Hoffman and Sachs : Introduction to the theory of plasticity for engineers
3. Mendelson : Plasticity theory and applications
4. Nadai : Theory of flow and fracture of solids
5. J. Chakraborty : Plasticity
6. S.K. Chakravorty : Mathematical theory of plasticity
7. Frager and Hodge : Theory of Perfectly plastic solid.

## **B – 1.13 : Differential Geometry and its Applications I (50 marks)**

Differentiable Manifold , Differentiable Curve , Tangent Space, Vector Field , Integral Curve, Differential of a mapping, One parameter group of transformations , Cotangent Space, r-form, Wedge Product, Exterior Product, Pull back differential form , Linear Connection, Torsion Tensor field , Curvature tensor field , Riemannian metric, Riemannian Connection , Riemannian manifold , Einstein manifold , Weyl conformal curvature tensor , Almost complex manifold , Complex manifold, Nijenhuis tensor , Hermite manifold , Almost contact manifold , contact manifold , Killing vector field , K- contact manifold

### References :

1. K.YANO,K.M.KON–Differentiable Manifold.
2. D.E.BLAIR–Contact Manifolds in Riemannian Geometry,Lecture Notes in Maths.
3. R.RESNIK–Introduction to Special Theory of Relativity.
4. D.S.LONDEN–An Introduction to Tensor Calculus,Relativity and Cosmology.
5. A.N.MATREEV–Mechanics and Theory of Relativity.

**B – 1.14 : Dynamic Meteorology and NWP I (50 marks)**

1 : Introduction

The physical foundation, units and dimensions.

2 : Thermodynamics of dry air

Composition of the atmosphere, variable of state, Equation of state of a perfect gas, the Universal gas constant, mixture of the gases, molecular weight of dry air, the first law of thermodynamics, specific heats of gases, adiabatic process, potential temperature, entropy and the second law of thermodynamics.

3 : Aerological diagrams

Properties, area equivalence, different kinds of TD diagrams, the emagram, the skew (T, -log p) diagram, the tephigram.

4 : Thermodynamics of water vapour and moist air

Equation of state for water vapour, specific heats of water substance, equation of state of moist air, virtual temperature, change of phase, variation of latent heat, with temperature, the Clausius-Clapeyron equation, adiabatic process for saturated air, moisture variables.

5 : Hydrostatic equilibrium

The hydrostatic equation, definition of lapse rate, the thickness equation pressure height formulae in model atmospheres, dry atmosphere with a constant lapse-rate, height and lapse rate of a homogeneous atmosphere the dry adiabatic atmosphere, the isothermal atmosphere the US standard atmosphere.

6 : Hydrostatic stability and convection

State, unstable and neutral equilibrium the parcel method, convective available potential energy, lapse rate for unsaturated air, lapse rate for saturated air.

7 : The equation of motion 1 : The Coriolis force

Introduction, motion as observed with reference to a fixed frame of coordinates, motion as observed in a rotating frame of coordinates, simple mathematical derivation of the Coriolis force.

8 : The equation of motion 2 : Derivation in various coordinates

The pressure gradient force, the spherical earth, inertia motion, The equation of motion, Derivation of the components of the Coriolis force from the law of conservation of angular momentum, Derivation of the equation of motion in plane coordinates from rotating axes, Derivation of equation of motion in rotating polar coordinates, Derivation of the three dimensional equations of motion in spherical coordinate system, Equation of motion in tangential curvilinear coordinates.

9 : Balanced flow

Introduction, geostrophic equation, effect of friction, the gradient wind equation, gradient wind solutions for the anticyclonic and cyclonic cases, comparison of geostrophic and

gradient wind values , the cyclostrophic wind, the ‘strong roots’ the non-geostrophic front, geostrophic front, Tropopause frontal theory.

Reference :

1. Dynamic Meteorology- A basic course : Adrian Gordon, Warwick Grace, Peter Schwerdtfeger, Ronald Byron-Scott.
2. Dynamical and Physical Meteorology : George J Haltiner, Frank Martin.
3. Introduction to Dynamical Meteorology : Holton.
4. Fundamental of atmospheric physics : Murray L Salby.

## **B – 1.15 : Dynamical Oceanography I (50 marks)**

### **Properties of Sea Water:**

The Equation of State. Quantities related to Density. Expansion Coefficient. Specific Heat. Potential Temperature. Speed of Sound. Freezing Point of Sea Water. Heat Balance of the Ocean. Surface Density Changes and the Thermohaline Circulation of the Ocean.

### **Properties of Fluid at Rest.**

Equation of State. Thermodynamic Variables. Balance of Forces in a Fluid at Rest. Static Stability.

### **Equations Satisfied by Moving Fluids.**

Mass Conservation Equation. Balance for a Scalar Quantity like Salinity. Internal Energy (or Heat) Equation. The Equation of Motion. Mechanical Energy Equation. Total Energy Equation. Bernoulli’s Equation. Adjustment under Gravity in a non-rotating System. Introduction : Adjustment to Equilibrium. Perturbation from the Rest State for a Homogeneous Inviscid Fluid. Surface Gravity Waves – Dispersion. Short-wave and Long-wave Approximations. Shallow Water Equations Derived using the Hydrostatic Approximation. Energetics of Shallow Water Motion. Sediments and tides in Channels and Gulfs.

### **Adjustment under Gravity of a Density Stratified Fluid.**

Introduction. The Case of two Superposed Fluids of Different Density. Baroclinic Mode and Rigid Lid Approximation. Adjustments within a Continuously Stratified Incompressible Fluid. Internal Gravity Waves, Dispersion, Energy of Internal Waves. Internal Waves Generated at a Horizontal Boundary. Free Waves in the Presence of Boundaries. Waves of Large Horizontal Scale: Resolution into Normal Modes for the Ocean.

### **Gravity Waves in a Rotating Fluid.**

Effects of Rotation on Surface Gravity Wave. Vertically Propagating Internal Gravity Wave in a Rotating Fluid. Energetics. Internal Wave Spectrum in the Ocean.

**References:**

1. O. M. Philips: Dynamics of the Upper Ocean. Cambridge University Press (1966)
2. J. Pedlosky: Geophysical Fluid Dynamics. Springer (1987)
3. G. P. Pickand: Descriptive Physical Oceanography. Oxford Pergaman Press (1975)
4. A. E. Gill: Atmospheric Ocean Dynamics. Academic Press (1982)

**B – 1.16 : Elastodynamics – I (50 Marks)**

Equations of motion in an elastic solid. Diletational and Distorsional waves in isotropic elastic media. Two-dimensional motion. Helmholtz decomposition and displacement potentials. Propagation of plane waves. Flux of energy in time harmonic plane progressive waves.

Antiplane and inplane pro

blems. Waves in infinite media. Problems involving wave propagation in infinite media. Harmonic dilatational wave from a spherical cavity. Strain energy function. Longitudinal waves in a thin elastic rod. Flexural vibration of thin rod. Transverse vibration of plates.

Waves in layered media. Surface waves. Raleigh waves. Love waves. Stonely waves.

**References :**

1. Elastodynamics, Vol.-II : A.C.Eringen and E.S.Suhubi.
2. Wave Motion in Elastic Solids : K.F.Graff.
3. Stress waves in Solids : H.Kolsky.
4. Wave Propagation in Elastic Solids : J.D.Achenbach.
5. Wave Motion : J.Billingham and A.C.King.
6. Elastic Wave Propagation : A.Bedford and D.S.Drumheller.
7. Elastic Waves in Layered Media : W.M.Ewing, W.S.Jardetzky and F.Press.
8. On Wave propagation in Elastic Solids with cracks : Ch. Zhang and D. Gross.
9. Linear and Nonlinear Waves : G.B.Whitham.

**B – 1.17 : General Theory of Relativity and Cosmology I(50 Marks)**

**Minkowski spacetime** : Past and future Cauchy development, Cauchy surface. DeSitter and anti-de Sitter space-times. Robertson-Walker spaces. Spatially homogeneous space-time models. The Schwarzschild and Reissner – Nordstrom solutions. Kruskal diagram. Causal structure. Orientability. Causal curves. Causality conditions. Cauchy developments. Global hyperbolicity. The existence of Geodesics. The Causal boundary of space-time. Asymptotically simple spaces.

**References :**

1. The large scale structure of space-time - Hawking and Ellis (Camb. Univ. Press).
2. General Relativity – R.M. Wald ( Chicago Univ. Press ).
3. A first course in general relativity – B.F. Schutz (Camb. Univ. Press).
4. Gravitation and Cosmology – S. Weinberg (J. Wiley and Sons).
5. General Relativity, Astrophysics and Cosmology – Raychaudhuri, Banerji and Banerjee (Springer-Verlag).
6. General Relativity – M. Luduigsen (Camb. Univ. Press).
7. Introducing Einstein’s Relativity – R d’Inverno (Clarendon Press, Oxford).

**B – 1.18 : Generalised Functions and Wavelet Theory I (50 Marks)**

**Generalised Functions - 50 marks**

Good function, Fairly good function, Generalised function, Ordinary function as generalised function, Addition of generalised function, Derivatives of generalised functions, Fourier transform of generalised functions, Limits of generalised function, Powers of  $|x|$  as generalised functions, Even and odd generalised functions, Integration of generalised functions, Integration of generalised function, Multiplication of two generalised functions.

**Reference:**

1. Generalised Functions, by D.S.Jones.

**B – 1.19 : Graph Theory I ( 50 Marks )**

**Fundamental Concepts** : Basic Definitions. Graphs, Vertex degrees, Walks, Paths, Trails, Cycles, Circuits, Subgraphs, Induced subgraph, Cliques, Components, Adjacency Matrices, Incidence Matrices, Isomorphisms. (10)

**Graphs with special properties** : Complete Graphs. Bipartite Graphs. Connected Graphs, k-connected Graphs, Edge-connectivity, Cut-vertices, Cut-edges. Eulerian Trails, Eulerian Circuits, Eulerian Graphs : characterization, Hamiltonian (Spanning) Cycles,

Hamiltonian Graphs : Necessary condition, Sufficient conditions (Dirac, Ore, Chvatal, Chvatal-Erdos), Hamiltonian Closure, Traveling Salesman Problem. ( 10 )

**Trees** : Basic properties, distance, diameter. Rooted trees, Binary trees, Binary Search Trees. Cayley's Formula for counting number of trees. Spanning trees of a connected graph, Depth first search (DFS) and Breadth first search (BFS) Algorithms, Minimal spanning tree, Shortest path problem, Kruskal's Algorithm, Prim's Algorithm, dijkstra's Algorithm. Chinese Postman Problem. ( 10 )

**Coloring of Graphs** : Vertex coloring : proper coloring, k-colorable graphs, chromatic number, upper bounds, Cartesian product of graphs, Structure of k-chromatic graphs, Mycielski's Construction, Color-critical graphs, Chromatic Polynomial, Clique number, Independent (Stable) set of vertices, Independence number, Clique covering, Clique covering number. Perfect graphs : Chordal graphs, Interval graphs, Transitive Orientation, Comparability graphs. Edge-coloring, Edge-chromatic number, Line Graphs.

**References :**

1. Introduction to Graph Theory, Douglas B. West, Prentice-Hall of India Pvt. Ltd., New Delhi 2003.
2. Graph Theory, F. Harary, Addison-Wesley, 1969.
3. Basic Graph Theory, K.R. Parthasarathi, Tata McGraw-Hill Publ. Co. Ltd., New Delhi, 1994.
4. Graph Theory Applications, L.R. Foulds, Narosa Publishing House, New Delhi, 1993.
5. Graph Theory with Applications, J.A. Bondy and U.S.R. Murty, Elsevier science, 1976.
6. Graphs and Digraphs, G. Chartrand and L. Lesniak, Chapman & Hall, 1996.
7. Theory of Graphs, O. Ore, AMS Colloq. 38, Amer.Math.Soc., 1962.
8. Graph Theory, R. Gould, Benjamin / Cummings, 1988.
9. Graph Theory, J. Gross and J. Yellen, CRC Press, 1999.
10. Graph Theory with Applications to Engineering and Computer Science, Narsingh Deo, Prentice-Hall of India Pvt.Ltd., New Delhi, 1997.

**B –1. 20: Information Theory and Coding I (50 Marks)**

**Information Theory (50) :**

Introduction and preview. Measure of Information. Axioms for a measure of uncertainty. The Shannon entropy and its properties. Relative entropy and mutual information. Joint entropy and conditional entropy, relationship between entropy and mutual information. Jensen's inequality and its consequences. The log sum inequality. Data processing inequality. Problems.

Data compression : Codes. Kraft's inequality. Optimal codes. Bounds on the optimal code length. Kraft's inequality for uniquely decodable codes. Instantaneous codes and construction of instantaneous codes.

Discrete memory less channel. Classification of channels. Information processed by a channel. Calculation of channel capacity. Cost function.

Continuous channels. The time-discrete Gaussian channel. The converse to the coding theorem for time-discrete Gaussian channel. The time-continuous Gaussian channel. Problems.

### References :

1. Elements of Information Theory : Thomas M. Cover and Joy A. Thomas.
2. The Theory of Information and coding : R.J. McEliece.
3. Information Theory, Inference and learning algorithms : David J.C. Kackay.
4. Information Theory : R. Ash.
5. An introduction to information theory : F.M. Reza.
6. Introduction to coding theory : J.H. Van Lint.
7. Coding Theory – The Essentials : D.G. Hoffman, D.A. Leonard, C.C. Lindner, K.T. Phelps, C.A. Rodger and J.R. Wall.
8. A first course in coding theory : R. Hill.
9. Error correcting coding theory : M.Y. Rhee.
10. Algebraic coding theory : E.R. Berlekamp.

## B – 1.21 : Magneto Fluid Mechanics I(50 Marks)

### I: Basic concepts of Electromagnetic theory :

Coulomb's law, electric field, Gauss flux theorem and Gauss law, Polarisation, electrostatic energy due to a system of charges, Fundamental equations of electrodynamics, the law of Biot and Savart, magnetic induction field, Lorentz force, Maxwell's electromagnetic field equations. Laws of electromagnetic induction : Faraday's law and Lenz's law, Ohm's law.

### II : Magneto Hydrodynamics

Equation of motion of a conducting fluid, simplification of MHD equations using dimensional consideration (i.e. MHD approximations), magnetic Reynold's number, Alfven's theorem, the magnetic body force, Ferraro's law of isoration, Non-dimensional form of the equation.

### III : Magneto Hydrostatics

Force-free magnetic fields, Pinches, Bennett pinch or linear pinch, instability of linear pinch, equilibrium of sunspots, Chandrasekhar's equipartition of energy theorem.

### IV : Few solutions on MHD flows

Steady laminar flow of a viscous conducting fluid between parallel walls in the presence of a transverse magnetic field (i.e. Hortmann flow), Two dimensional MHD equations, Couette flow, Transient Couette flow, steady flow of a conducting liquid through a pipe of rectangular cran section in the presence of a transverse magnetic field, unsteady flow of a conducting liquid near an infinite flat plate in the presence of a transverse magnetic flow.

**References:**

- 1) An introduction to Magneto-Fluid Mechanics-V, C.A>Ferraro & C. Plumpton.
- 2) Magneto Hydrodynamics – T.G. Cowling.
- 3) Text Book on Fluid Dynamics – F. Chorlton.
- 4) Magneto-Fluid Dynamics for Engineers and Physicists – K.R. Cramer & S.I. Pai.
- 5) Magneto Gas dynamics & plasma dynamics – S.I. Pai. Home

**B – 1.22 : Mathematical Ecology I (50 Marks)**

The nature of ecosystems, Autotroph-based ecosystem, Detritus-based ecosystem, Different types of population growth, Community dynamics- succession and community responses.

**Single Species Population Dynamics:**

Continuous growth models – their stability analysis, Influence of random perturbations on population stability. Insect out break model- Spruce-Budworm model. General autonomous models. Delay Models

**Population Dynamics of Two Interacting Species:**

Introduction, Lotka-Volterra system of predator-prey interaction, Trophic function, Gauss's Model, Gause Model, Kolmogorov Model, Leslie Gower Model, Analysis of predator-prey model with limit cycle periodic behavior, parameter domains of stability. Competition models- exclusion principle and stability analysis. Models on mutualism.

**Continuous models for three or more interacting species:**

Three species simple and general food chain models- its stability and persistence. Models on one prey two competing predators with limited resources and living resource supporting three competing predators- stability analysis and persistence.

**Biological Models Using Linear and Nonlinear Difference Equations:**

Nicholson-Bailey model, modification of Nicholson-Bailey model (density dependence in the host population), model for plant-herbivore interaction and its stability analysis.

**References:**

1. H. I. Freedman - Deterministic Mathematical Models in Population Ecology
2. Mark Kot (2001): Elements of Mathematical Ecology, Cambridge Univ. Press.
3. D. Alstod (2001): Basic Populus Models of Ecology, Prentice Hall, Inc., NJ.

**B – 1.23: Mathematical Statistics I (50 Marks)**

Review of the concepts : Population, Sample, Parameter, Statistic, Sampling Distribution.

Estimation of parameters. Unbiasedness. Consistency. Sufficiency. Cramer-Rao bound. Rao-Blackwell theorem. Efficiency. Maximum likelihood estimators. Properties. Case of several parameters.

Tests of statistical hypotheses. Simple and composite hypothesis. Size and power of a test. Neyman-Pearson lemma and UMP test. Unbiasedness and similarity. UMPU and UMPS tests.



Gauss-Markoff setup and least squares estimation. LS estimation with restriction on parameters. Simultaneous estimation of parametric functions.

Multivariate normal distribution. Hotelling  $T^2$ .

**References:**

1. Rao, C.R. – Linear Statistical Inference and its Applications.
2. Wilks, S.S. – Mathematical Statistics.
3. Ferguson, T.S. – Mathematical Statistics.

**B – 1.24 : Measure and Topology I (50Marks)**

**Integration:**

Signed Measure, Hahn Decomposition Theorem, Mutually Singular Measures, Radon-Nikodym Theorem, Lebesgue Decomposition, Riesz Representation Theorem, Extension Theorem (Caratheodory), Lebesgue Stieljes Integral, Product Measure, Fubini's Theorem, Differentiation and Integration, Decomposition into Absolutely Continuous and Singular Parts.

Measure on Locally Compact Spaces: Borel Sets, Baire Sets, Baire Sandwich Theorem, Borel and Baire Measure, Regularity of Measures, Regular Borel Extension of a Baire Measure, Completion, Continuous Functions with Compact Support, Integration of Continuous Functions with Compact Support, Riesz-Markoff Theorem.

**References :**

1. Halmos, P.R., *Measure Theory*, Van Nostrand, Princeton, 1950.
2. Oxtoby, J.C., *Measure and Category*, Springer Verlag, 1980.
3. Berberian, S.K., *Measure and Integration*, Chelsea Publishing Company, NY, 1965.
4. Barra, G.de, *Measure Theory and Integration*, Wiley Eastern Ltd., 1981.
5. Rana, I.K., *An Introduction to Measure and Integration*, Narosa Publishing House, Delhi, 1997
6. Bartle, R.G., *The Elements of Integration*, John Wiley and Sons, Inc., NY, 1966.

**B – 1.25: Mechanics of Viscous fluids and Boundary layer Theory I  
(50 Marks)**

**Mechanics of viscous fluids –**

Viscous fluids, velocity strain – tensor and stress-strain relations for viscous fluids (statement of relations only). The Navier – Stokes equations of motion in Cartesian Coordinates and statements of its equivalent forms in spherical, polar and cylindrical Coordinates. Dissipation of energy due to viscosity, steady motion between parallel planes, Theory of lubrication, steady motion in a tube of different cross-sections. Vorticity in viscous fluids, Circulation in viscous fluids. Diffusion of vorticity, steady flow past a

fixed sphere, Dimensional Analysis, Reynolds number, Steady motion of a viscous fluid due to a slowly rotating sphere.

### Two Dimensional Motion

Equation satisfied by the stream function for a motion under conservative field of external forces, Hamel's equation, Logarithmic spirals,

### Three Dimensional Motion

Stokes' solution for a slow steady parallel flow past a sphere, stream function and the flow pattern, Oseen's criticism, Oseen's solution for slow steady parallel flow past a sphere and past a circular cylinder.

### References :

1. Theoretical Hydrodynamics --- Milne – Thomson
2. Viscous flow Theory --- S.I. Pai
3. Hydrodynamics --- H. Lamb
4. A Treatise on Hydrodynamics Part II -- W.H. Besant & A.S. Ramsay
5. Text book of Fluid Dynamics --- F. Chorlton

### B – 1.26 : Non-linear and Dynamic Programming I (50 Marks)

Unconstrained and equality-constrained extremum. First order and second order necessary conditions. Sufficient conditions. Working principle for testing optimality.

Optimality with equality constraints only and with both equality and inequality constraints. Farkas' lemma (Forms I and II). Theorem of the alternative (Forms I and II). Fritz-John necessary conditions. Kuhn-Tucker first and second order necessary conditions. Sufficient condition. Working procedure for testing optimality. Saddle point of the Lagrangian and its relation to optimality.

Convex programming. Necessary and Sufficient condition. Generalized Kuhn-Tucker saddle point theorem.

Quadratic programming. Necessary and Sufficient condition for optimal solution.

### References :

1. Viscous flow theory, Vol.I --- S.I. Pai
2. Hydrodynamics --- H. Lamb
3. New methods in laminar boundary layer theory --- D. Meksyn.
4. Elementary treatise on hydrodynamics and sound --- A.B. Besset.
5. Modern developments in fluid dynamics --- S. Goldstein.
6. Boundary layer theory --- H. Schlichting.
7. Laminar boundary layers --- L. Rosenhead
- 8.

### B -1. 27 : Number Theory : 50 Marks

Revision of Unique Factorization, Congruencies, Chinese Remainder Theorem.

Structure of  $U(\mathbb{Z}/n\mathbb{Z})$ , Quadratic Reciprocity, Quadratic Gauss Sum, Finite Fields, Gauss and Jacobi Sums, Cubic and Biquadratic Reciprocity, Equations over Finite Fields, Zeta

Function.

Algebraic Number Fields and the Ring of Integers, Units and Primes, Factorisation, Quadratic and Cyclotomic Fields, Dirichlet L Function, Diophantine Equations, Elliptic Curves.

### References :

1. Ireland & Rosen, *A Classical Introduction to Modern Number Theory*, Springer.
2. Niven, I., Zuckerman, S.H., Montgomery, L.H., *An Introduction to the Theory of Numbers*, Wiley.
3. Serre, J.-P., *A Course in Arithmetic*, Springer.
4. Cassels, J.W.S., Frolich, A., *Algebraic Number Theory*, Cambridge.

## B – 1.28. Operating Systems (50 Marks)

**Theory - 30, Assignment - 20** (Computer lab access is necessary and mandatory)

Introduction: Basic architectural concepts, interrupt handling, concepts of batch-processing, multiprogramming, time-sharing, real-time operations; Resource Manager view, process view and hierarchical view of an OS. 4 lectures

Processor management: CPU scheduling - short-term, medium term and long term scheduling, non-preemptive and preemptive algorithms, performance analysis of multiprogramming, multiprocessing and interactive systems; Concurrent processes, precedence graphs, critical section problem – 2-process and n-process software and hardware solutions, semaphores; Classical process co-ordination problems, Producer consumer problem, Reader-writer problem, Dining philosophers problem, Barber's shop problem, Inter-process communication. 20 lectures

Concurrent Programming: Critical region, conditional critical region, monitors, concurrent languages, concurrent pascal, communicating sequential process (CSP);

15 lectures

Deadlocks: prevention, avoidance, detection and recovery. 15 lectures

Memory management: Partitioning, paging, concepts of virtual memory, demand-paging – page replacement algorithms, working set theory, load control, segmentation, segmentation and demand paging, Cache memory management. 12 lectures

Device Management: Scheduling algorithms - FCFS, shortest-seeK-time-first, SCAN, C-SCAN, LOOK, C-LOOK algorithms, spooling, spool management algorithm.

8 lectures

Information Management: File concept, file support, directory structures, symbolic file directory, basic file directory, logical file system, physical file system, access methods, file protection, file allocation strategies. 8 lectures

Protection: Goals, policies and mechanisms, domain of protection, access matrix and its implementation, access lists, capability lists, Lock/Key mechanisms, passwords, dynamic protection scheme, security concepts and public and private keys, RSA algorithms.

8 lectures

A case study: UNIX OS file system, shell, filters, shell programming, programming with the standard I/O, UNIX system calls. 10 lectures

### References:

1. A. Silberschatz and P. B. Galvin: Operating Systems Concepts, 5th ed., John Wiley and Sons, New York, 1998.
2. A. S. Tannenbaum: Modern Operating Systems, Prentice Hall, Englewood Cliffs, 1992.
3. S. E. Madnick and J. J. Donovan: Operating Systems, McGraw Hill, New York, 1974.

## **B –1. 29 : Operator Algebra I (50 Marks)**

### **Banach Algebra**

Definition of Banach Algebra and examples, Singular and Non-singular elements, The spectrum of an elements, The spectral radius, Gelfand formula, Multiplicative linear functionals and the maximal ideal space, Gleason-Kahane-Zelazko Theorem, The Gelfand Transforms, The Spectral mapping theorem, Isometric Gelfand Transform.

### **C\* - Algebras :**

Definition of C\*-Algebras and examples, Self-adjoint, Unitary, Normal, Positive and Projection elements in C\*-Algebras, Commutative C\*-Algebras, C\*-Homomorphisms, Representation of commutative C\*-Algebras, subalgebras and the spectrum, The Spectral Theorem, Positive linear functionals in C\*-algebras, States and the GNS construction.

### **References :**

1. Bonsall and Duncan, Complete Normed algebras, Springer-Verlag.
2. Kadison and Ringrose, Fundamentals of operator theory, Vol. I and II, Academic press.
3. Rickart, General theory of Banach Algebras, : Van Nostrand.
4. W. Arveson, An invitation to C\*-Algebras, Springer-Verlag.
5. Palmer, Banach Algebras and the general theory of C\*-algebras, Cambridge University Press.

## **B – 1.30: Operator Theory I (50 Marks)**

### **Unit 3.3 Bounded linear Operators: (50 Marks)**

Resolvent set, Spectrum, Point spectrum, Continuous spectrum, Residual spectrum, Approximate point spectrum, Spectral radius, Spectral properties of a bounded linear operator, Spectral mapping theorem for polynomials. Numerical range, Numerical radius, Convexity of numerical range, Closure of numerical range contains the spectrum, Relation between the numerical radius and norm of abounded linear operator A()

### **Banach Algebra:**

Definition of normed and Banach Algebra and examples, Singular and Non-singular elements, The spectrum of an element, The spectral radius.

### **Compact linear operators:**

Spectral properties of compact linear operators on a normed linear space, Operator equations involving compact linear operators, Fredholm alternative theorem, Fredholm alternative for integral equations. Spectral theorem for compact normal operators.

Reference Books :

1. Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley and sons.
2. G. Bachman and L. Narici, Functional Analysis, Dover Publications.
3. A . Taylor and D. Lay, Introduction to Functional Analysis, John Wiley and Sons.
4. N. Dunford and J.T. Schwartz, Linear Operators – 3, John Wiley and Sons.
5. P.R. Halmos, Introduction to Hilbert space and the theory of Spectral Multiplicity, Chelsea Publishing Co., N.Y.

### **B – 1.31: Plasma Mechanics I (50 Marks)**

Definition of Plasma as an ionized gas. Saha's equation of ionization. Occurrence of plasma in nature. Plasma as mixture of different species of charged particles.

Elements of kinetic theory (Statistical approach), Single particle phase space, Volume elements

Distribution function, Characterization of plasma with respect to the nature of the distribution function : Homogeneous, Inhomogeneous, Isotropic, Anisotropic

Derivation of Boltzmann equation, Average values and Macroscopic variables, Derivation of Macroscopic equations (Moment equations): Equation of continuity, Equation of motion, Equation of energy, Assumption on the nature of the distribution function to form a closed and consistent system of macroscopic equations (Equation of State), Cold Plasma limit, The equilibrium state: Maxwellian Distribution, Debye Shielding, The plasma parameter and the criteria for plasma formation,

Plasma-Single fluid approach (MHD): Approximation for MHD, Basic equations for MHD:, Conservation of mass, Conservation of momentum, Conservation of energy, Conservation of magnetic flux, Frozen-in-effect, Alfven theorem, Generalized Ohm's law, MHD equilibrium, The theta Pinch, The Z – Pinch, Axisymmetric toroidal equilibria , Linear Stability, The energy principle of ideal MHD, The Rayleigh – Taylor instability

First order orbit theory (Single particle motion ): Uniform **E** and **B** fields, Larmor orbits and guiding centers, The magnetic moment and the magnetization current, Non-uniform **B** field, Non-uniform **E** field, Time varying **E** field, Time varying **B** field, Adiabatic invariants,

**References:**

1. Plasma Physics and controlled Fusion, F.F. Chen, PLENUM PRESS, NEWYORK AND LONDON.
2. Fundamental of Plasma Physics, J.A. Bittencourt, PERGAMON PRESS, NEWYORK AND LONDON.

Theory of Plasma waves, T.H. Stix, McGraw Hill.

### **B – 1.32 : Probability and Stochastic Processes I (50Marks)**

Fields and  $\sigma$ -fields. Probability as a measure. Random variables. Probability distribution. Expectation. Moments. Absolute continuity and density function. Characteristic function. Inversion theorem. Convergence in probability. Weak convergence and continuity theorem for characteristic functions. Weak and strong law of large numbers. Law of the iterated logarithm. Lindeberg-Levy central limit theorem. Central limit theorem with Lindeberg and Liapunoff conditions.

Definition and classification of stochastic processes.

Simple random walk and gambler's ruin problem. Probability of ultimate ruin. Expected duration of game.

#### **References:**

1. Bhat, B.R. – Modern Probability Theory.
2. Chung, K.L. – Elementary Probability Theory and Stochastic Processes.
3. Billingsley, P. – Probability and Measure.
4. Srinivasan, S.K. and Mehata, K.M. – Stochastic Processes.
5. Hoel, P.G., Port, S.C. and Stone, C.J. – Introduction to Stochastic Processes.

### **B –1. 33 : Production and Inventory Control I (50 marks)**

Historical origin and background of the subject. Nature of inventory problems. Structure of inventory systems. Definition of inventory problem. Important parameters associated with inventory problems. Variables in inventory problems. Controlled and uncontrolled variables. Types of inventory systems and inventory policies. Statistical and dynamical inventory problems.

Deterministic inventory models / systems. Harris-Wilson model. Economic lot size systems. Sensitivity of the lot size systems. Order level systems and their sensitivity analysis. Order level lot size and their sensitivity studies. Non-constant demand models under  $(s, q)$ ,  $(t, s_i)$  and  $(t_i, s_i)$  policies. Power law and linear travel demand situations. Lot size systems with different cost properties: (i) Quantity discounts, (ii) Price-change anticipation, (iii) Perishable goods system.

Multi-item inventory models with (i) single linear restriction, (ii) More than one linear restrictions, (iii) non-linear restrictions.

#### **References:**

Inventory Systems : Eliczer Nadder – John Wiley and Sons.

Analysis of Inventory Systems : G. Hadley and T. M. Whitin – Prentice Hall.  
Principles of Inventory and Material Management : R. J. Tersine – North-Holland.

### **B- 1.34 : Quantum Field Theory and Statistical Mechanics I (50 marks)**

#### **Quantum Field Theory**

Lagrangian formation, symmetries and gauge fields, Real and complex regular fields. Noether's theorem. Topology and the vacuum: the Bohm-Aharonov effect. Yang-Mills field Canonical quantization and particle interpretation – real scalar field, complex scalar field and electromagnetic field. Path integrals and quantum mechanics. Path integral quantization. Feynman rules. Renormalization.

#### **References:**

1. L.D Landau and E.M Lifshitz: "The Classical Theory of Fields" (Oxford, Pergamen Press).
2. L.H Ryder: "Quantum Field Theory" (Academic Publishers).
3. C. Itzykson and j.b Zuber: "Quantum Field Theory" (Mc Graw Hill).
4. J.D Bjorken and S.D Drell: "Relativistic Quantum Fields" (Mc Graw Hill).
5. F. Mandi: " Introduction to Quantum Field Theory" (Interscience, 1960).
6. R.P Feynman and A.R Hibbs, "Quantum Mechanics and Path Integrals" (Mc Graw Hill).

### **B – 1.35 : Queuing Theory and Game Theory I (50 Marks)**

Birth-and-death queuing models. Erlang's formula. Finite-source queues. State-dependent service. Impatience. Haight's models. Balking and reneging.  
Bulk-input model. Bulk=arrival model. Models with Erlangian arrival; or service.  
M/G/1 models. Pollackzek-Kninchin formula. Departure-point steady-state analysis. Proof of necessary and sufficient condition for ergodicity. Waiting times. Busy period.  
Bulk-input model  $M^{(X)}/G/1$ . Departure state dependence. Generalized Little's formula for M/G/c. Proof of Erlang's formula for GI/G/c.  
Transient analysis for M/G/1 and GI/M/1 models. Steady state arrival-point analysis. Generalization to. GI/M/c.

#### **References:**

8. Gross and Harris – Fundamentals of Queuing Theory.
9. Saaty – Elements of Queuing Theory.
10. Chaudhury and Kashyap – Introduction to of Queuing Theory.

## **B – 1.36: Renewable Bio-Economic Modelling (50 Marks)**

### **Elementary dynamics of exploited populations:**

The logistic growth models, constant rate harvesting, fishing effort, generalized logistic model's depensation, Yield-effort curves, critical depensation

### **Open-Access Fishery:**

Gordon's static model, opportunity cost, externality, economic over fishing, production function, Cobb-Douglas production function, discounting, Schaefer model: optimal harvesting policy, effect of discounting.

### **Elements of Control Theory:**

One-dimensional control problem, linear variational problem-singular path, blocked interval, impulse control

### **The Maximum Principle and Its Application in Linear Variational Problem:**

Transversality condition, feedback control, Pontryagin's maximal theory, its economic interpretation, structure of the multidimensional optimal control problem and the maximum principle, growth and aging, Beverton-Holt fishery model, dynamic optimization.

### **Multi-species models in fishery management:**

Combined harvesting of two ecologically independent fish species following logistic growth, bionomic equilibrium optimal harvest policy, combined harvesting of two competing fish species following logistic growth.

### **Forestry Management:**

The Faustmann model, Kilkki-Vaisanen model, matrix model for the management of a height structured forest, modeling on degradation and subsequent regeneration of forestry resources.

### **References:**

1. C. W. Clark, (1976): *Mathematical Bioeconomics: The optimal Management of Renewable Resources*, John Wiley & Sons, New York.
2. C. W. Clark, (1985): *Bioeconomics Modeling and Fisheries Management*, John Wiley & Sons, New York.
3. B. S. Goh (1980): *Management and Analysis of Biological Populations*, Elsevier, Amsterdam.



## **B – 1.37 : Theory of Marketing Decisions I (50 marks)**

Elements of control theory, Basic optimal control problem, Augmented functional, Hamiltonian function, Adjoint equation, Transversality condition, Pontryagin's maximum principal (formulation only) with continuous and discontinuous controls, Applications.

Basic concepts of demand curve, Supply curve, Price elasticity of demand and supply, Utility of consumption and consumers surplus, Different market forms of pure computations, Competitive equilibrium, Monopoly, Multi-product monopoly, Price discrimination, Oligopoly, Conjectural variation. Theory of costs of production, Marginal costs, Relationship between marginal productivity, average cost and marginal cost.

Theory of production, Production function, Cobb-Douglas production function and its properties, Elasticity of substitution, Constant production function, Derivation of the cost function from the production function – The Cobb-Douglas case, Solow-Minhaus-Arrow-Chenery production function and its properties.

### **References:**

1. Mathematical Analysis for Economists – R. G. D. Allen : McMillan.
2. Mathematical Economics – R. G. D. Allen : McMillan.
3. Mathematical Formulation of Micro-economists – M. M. Metwally: Asia Publishing House.
4. Quantitative Techniques for Marketing Decisions – Marvin A. Jolson and Richard T. Hisc : McMillan.
5. Marketing Management – Philip Kotler : Prentice Hall.

## **B – 1.38: Theory of Semigroups I (50 Marks)**

**Introduction:** Basic Definitions. Monogenic semigroups, Periodic semigroups. Ordered sets, semilattices and lattices. Binary relations, equivalences. Congruences. Ideals and Rees congruences. (20)

**Green's Equivalences:** Green's equivalence relations:  $\square, \square, \square, \square$  and  $\square$ . The structure of  $\square$ - classes, Regular  $\square$ -classes, Regular semigroups. (10)

**Completely regular semigroups:** Characterization of completely regular semigroups as union of groups, Semilattices of groups, Clifford semigroups, Orthodox semigroups. (20)

### References:

1. Fundamentals of semigroup theory, J. M. Howie, Clarendon Press, oxford, 1995.
2. An introduction to semigroup theory, J. M. Howie, Academic Press, London, 1976.

3. The algebraic theory of semigroups, Amer. Math. Soc., Math Surveys No. 7, Providence, Vol I, 1961, Vol II, 1967.
4. Completely Regular Semigroups, M. Petrich and N. R. Reilly, John Wiley & Sons.
5. Introduction to semigroups, M. Petrich, Merrill, Columbus, 1973.
6. Structure of regular semigroups, M. Petrich, Univ. de Montpellier, 1977.
7. Lectures in semigroups, M. Petrich,
8. Semigroups and combinatorial applications, G. Lallement,
9. Completely 0-simple semigroups: an abstract treatment of the lattice of congruences, Benjamin, New York, 1969.
10. A course in universal algebra, S. Burris and H. P. Sankappanavar, Springer, New York, 1981

### **B – 1.39 : Topological Groups and Harmonic Analysis I : 50 Marks)**

Definition of a topological group and its basic properties, Subgroups and Quotient Groups, First, Second, Third Isomorphism Theorems, Properties of Topological Groups involving Connectedness, Separation Axioms in topological groups, Invariant Pseudo Metrics, Uniform Structure in Topological Groups, Compact and Locally Compact Topological Groups

Preliminaries of Lebesgue Measure, Measurable Function, Integration, Product Measure, Fubini's Theorem, Signed Measure, Hahn Decomposition Theorem, Mutually Singular Measure, Radon Nikodym Theorem, Lebesgue Decomposition, Decomposition into Absolutely Continuous and Singular parts, Differentiation and Integration. Baire and Borel Sets, Baire Sandwich Theorem, Borel and Baire Measures on Locally Compact Spaces, Regularity of Measure, Integration of Continuous Functions with compact Support, Riesz-Markoff Theorem.

#### **References:**

1. Hewitt, E., Ross, K., *Abstract Harmonic Analysis*, Vol I, Academic Press, NY, 1963.
2. Bachman, G., *Elements of Abstract Harmonic Analysis*, Academic Press, NY and London, 1964.
3. Rudin, W., *Fourier Analysis on Groups*, McGraw Hill Publishing Co. Ltd.
4. Loomis, L., *An Introduction to Abstract Harmonic Analysis*, Van Nostrand, N.J., 1953.
5. Goldberg, R.R., *Fourier Transforms*, Cambridge University Press, London & NY, 1961.

## **Second year Second Semester (250 Marks)**

### **Unit 4.1: Advanced Numerical Analysis (Theory : 20, Practical : 30)**

#### **Numerical solution of integral equations :**

Approximate solution of Fredholm equation by finite sums and degenerate Kernels.  
Numerical approximation of Volterra equations.

#### **Finite element and Boundary element methods :**

Weighted residual method : Galerkin, Least square, partition, moment and collocation methods. Solution of boundary value problems by Ritz method. Finite elements and boundary elements of various terms. Constant elements by Gaussian quadrature. Numerical integration over finite elements. Solution of b.v. problems by Finite element and Boundary element methods.

#### **References :**

1. Computing methods ; Berzin and Zhidnov.
2. Analysis of Numerical methods : Isaacson and Keller.
3. A first course in Numerical Analysis : Ralston and Rabinowitz.
4. Numerical solution of differential equations : M.K.Jain.
5. Numerical solution of partial differential equations : G.D.Smith.
6. The finite element method in structural and continuum mechanics : O.C.Zienkiewics.
7. The finite elements method in partial differential equations : A.R.Mitchell.
8. An introduction to boundary element methods : Prem K. Kytbe.
9. Computational Mathematics : B.P.Demidovich and J.A.Maron.
10. Applied Numerical Methods : A. Gourdin & M. Boumahrat.

#### **List of Practical Problems**

1. Solution of ordinary differential and partial differential equation by weighted Residual method :
  - a. Least square method.
  - b. Galerkin method.
2. Solution of simple boundary value problem by
  - a. Finite element and

- b. Boundary element method.
- 3. Solution of system of Non-linear equations by Newton's method.
- 4. Method of steepest descent.

## Unit 4.1 Advanced Functional Analysis

Examples of Banach Spaces, Stone-Weierstrass Theorem, Ascoli-Arzelà Theorem.  $L^p$ -spaces, Completeness and other Properties. Linear Topological Spaces, Locally Convex Spaces and their Characterization in terms of a family of Seminorms, Hahn-Banach Theorem, Separation Theorem, Open Mapping Theorem, Closed Graph Theorem, Weak Topology and Duality Theorem for Normed Linear Spaces, Krein-Milman Theorem and its Applications, Uniform Convexity, Strict Convexity and their Applications. 0.25in

### References :

Kelley, J.L., Namioka, I., *Linear Topological Spaces*, D.Van Nostrand Company, 1963.  
 Rudin, W., *Functional Analysis*, Tata McGraw-Hill Publishing Corpn. Ltd., 1979.  
 Aliprantis, C.D., Burkinshaw, O., *Principles of Real Analysis*, 3rd Edition, Harcourt Asia Pte Ltd., 1998.  
 Goffman, C., Pedrick, G., *First Course in Functional Analysis*, Prentice Hall of India, New Delhi, 1987.  
 Taylor, A.E., *Introduction to Functional Analysis*, John Wiley and Sons, New York, 1958.  
 Conway, J.B., *A Course in Functional Analysis*, Springer Verlag, New York, 1990.

## Unit 4.1: Discrete Mathematics – II

### The Foundations: Logic and Proofs : (15 marks)

Propositional Logic, Propositional Equivalences, Predicates and Quantifiers, Nested Quantifiers, Rules of Inference, Introduction to Proofs, Proof methods and strategy.

### Introduction to Combinatorics : (35 marks)

The Mathematics of Choice : The fundamental counting principle, Pascal's triangle, Error-correcting codes, Combinatorial identities, Four ways to choose, The binomial and multinomial theorems, Partitions, Elementary symmetric functions.

Recurrence : Some examples, The auxiliary equation method, Generating functions, Derangements, Catalan numbers.

Partitions and Colorings : Partitions of a Set, Stirling Numbers, Counting Functions, Vertex Colorings of Graphs, Edge Colorings of Graphs.

The Inclusion-Exclusion Principle : The principle, Counting surjections, Counting labeled trees, Scrabble, The Menage problem.

Latin squares and Hall's theorem : Latin squares and orthogonality, Magic squares, Systems of distinct representatives, From Latin squares to Affine planes.

Schedules and 1-Factorizations : The circle method, Bipartite tournaments and 1-factorizations of  $K_{n,n}$ , Tournaments from orthogonal Latin squares.

Introduction to designs : Balanced incomplete block designs, Resolvable designs, Finite projective planes, Hadamard matrices and designs, Difference methods, Hadamard matrices and codes.

**Text Books :**

1. Discrete Mathematics, Kenneth H. Rosen, Mc-Graw Hill, International Edition 2007.
3. A First Course in Discrete Mathematics, Ian Anderson, Springer, 2000

**References :**

1. Combinatorics, Russell Merris, Wiley-Interscience, 2003.
2. Discrete Mathematics for Computer Scientists & Mathematicians, Joe L. Mott, Abraham Kandel and Theodore P. Baker, Prentice-Hall of India Pvt. Ltd., 2003.
3. Applied Combinatorics, Fred S. Roberts, Prentice-Hall of India Pvt. Ltd.
4. H. Enderton: A Mathematical Introduction to Logic, Academic Press, London, 1972.

## **Unit 4.2 Integral Equation and Integral Transform**

### **Integral Equations – 25 marks**

1. Linear integral equations of 1<sup>st</sup> and 2<sup>nd</sup> kinds – Fredholm and Volterra types. Relation between integral equations and initial boundary value problems . Integral equations.
2. Existence and uniqueness of continuous solutions of Fredholm and Volterra's integral equations of second kind. Solution by the method of successive approximations. Iterated kernels. Reciprocal kernels. Volterra's solution of Fredholm's integral equation.
3. Fredholm theory for the solution of Fredholm's integral equation. Fredholm's determinant  $D(\lambda)$ . Fredholm's first minor  $D(x,y,\lambda)$  Fredholm's first and second fundamental relations. Fredholm's  $p$ -th minor. Fredholm's first, second and third fundamental theorems. Fredholm's alternatives.
4. Hilbert-Schmidt theory of symmetric kernels. Properties of symmetric kernels. Existence of characteristic constants. Complete set of characteristic constants and

complete orthonormalised system of fundamental functions. Expansion of iterated kernel  $k_n(x,t)$ , in terms of fundamental functions. Schmidt's solution of Fredholm's integral equations.

5. Applications

**Integral Transforms – 25 marks**

1. Fourier Transforms: Fourier integral Theorem. Definition and properties. Fourier transform of the derivative. Derivative of Fourier transform. Fourier transforms of some useful functions. Fourier cosine and sine transforms. Inverse of Fourier transforms. Convolution. Properties of convolution function. Convolution theorem. Applications.
2. Laplace transforms: Definition and properties. Sufficient conditions for the existence of Laplace Transform. Laplace Transform of some elementary functions. Laplace transform of the derivatives. Inverse of Laplace transform. Bromwich Integral Theorem. Initial and final value theorems. Convolution theorem. Applications.
3. Mellin and Hankel Transforms and their inverse. Application to Boundary value problems.
4. Z-transform : Definition and properties. Z-transform of some standard functions. Inverse Z-transforms. Applications.

**References :**

1. I.N. Sneddon : The Uses of Integral Transforms.
2. C.J. Tranter : Integral Transforms.
3. I.N. Sneddon : Fourier Transform.
4. Lovitt : Linear Integral Equations.
5. Tricomi : Integral Equations.
6. Andrews & Shivamoggi : Integral transforms for Engineers.
7. L. Debnath & D. Bhatta : Integral Transforms and Their Applications.

Unit 4.3 One course from A-2.1 to A-2.8

(\*corresponding to the course selected in Unit 3.3 in the 2<sup>nd</sup> year 1<sup>st</sup> semester)

A - 2.1	:	Advanced Algebra – II
A - 2.2	:	Differential Topology
A - 2.3	:	Introduction to Algorithms - II
A - 2.4	:	Fluid Mechanics II
A - 2.5	:	Mathematical Modelling of Biological Systems II
A - 2.6	:	Mathematical Theory of Elasticity II
A - 2.7	:	Principles of Operations Research II
A - 2.8	:	Quantum Mechanics

(\*If a student has studied A-1.4 in the 2<sup>nd</sup> year 1st semester then the student has to go for A-2.4 in this semester.)

**A – 2.1 : Advanced Algebra - II ( 50 Marks )**

**Multilinear Algebra (10 Marks) :**

Determinants, Tensor Algebras, Symmetric Algebras, Exterior Algebras, Homomorphisms of Tensor Algebras, Symmetric and Alternating Tensors.

**Note :** This course is based on the book [1]; Chapter 11.

**Structure of Rings (30 Marks) :**

Artinian rings, Simple rings, Primitive rings, Jacobson density theorem, Wedderburn - Artin theorem on simple (left) Artinian rings.

The Jacobson radical, Jacobson semisimple rings, subdirect product of rings, Jacobson semisimple rings as subdirect products of primitive rings, Wedderburn - Artin theorem on

Jacobson semisimple (left) Artinian rings.

Simple and Semisimple modules, Semisimple rings, Equivalence of semisimple rings with Jacobson semisimple (left) Artinian rings, Properties of semisimple rings, Characterizations of semisimple rings in terms of modules.

**Note :** This course is based on the books [4] and [3]; Chapter XVII.

### **Group Representations (10 Marks) :**

Representations, Group-Rings, Maschke's Theorem, Character of a Representation, Regular Representations, Orthogonality Relations, Burnside Two-Prime Theorem.

**Note :** This course is based on the book [3]; Chapter XVIII.

### **References :**

Dummit, D.S., Foote, R.M., *Abstract Algebra*, Second Edition, John Wiley & Sons, Inc., 1999.

Atiyah, M., MacDonald, I.G., *Introduction to Commutative Algebra*, Addison-Wesley, 1969.

Lang, S., *Algebra*, Addison-Wesley, 1993.

Lam, T.Y., *A First Course in Non-Commutative Rings*, Springer Verlag.

Hungerford, T.W., *Algebra*, Springer.

Jacobson, N., *Basic Algebra, II*, Hindustan Publishing Corporation, India.

Malik, D.S., Mordesen, J.M., Sen, M.K., *Fundamentals of Abstract Algebra*, The McGraw-Hill Companies, Inc.

Curtis, C.W., Reiner, I., *Representation Theory of Finite Groups and Associated Algebras*, Wiley-Interscience, NY.

## **A – 2.2 : Differential Topology (50) Marks**

**Smooth Mappings:** Inverse Function Theorem, Local Submersion Theorem ( Implicit Function Theorem).

**Differentiable manifolds:** Differentiable manifolds and submanifolds, examples, including surfaces,  $S^n$ ,  $RP^n$ ,  $CP^n$  and lens spaces, tangent bundles; Sard's theorem and its applications. Differentiable transversality, orientation, Whitney's embedding theorems. Pontryagin-Thom construction, Fruedenthal suspension theorem.

**Vector fields and differential forms :** Integrating vector fields, degree of a map, Brouwer Fixed Point Theorem, Poincare-Hopf Theorem, differential forms, deRham's theorem.

### **References :**



**Text :** Hirsch, Moris W. “ Differential Topology” Graduate Texts in Mathematics. Vol.33, Reprint edition. New York, Springer-Verlag, 1988.

Spivak : Calculus on Manifolds, Benjamin, 1965 ( differentiation, Inverse Function Theorem, Stokes Theorem )

Milnor : Topology from the Differentiable viewpoint, University of Virginia Press, 1965

James R Munkres : Elementary Differential Toplogy, 1966

J.W.Milnor : Differential Topology.

Guillemen Pollack : Differential Toplogy, Prentice-Hall, 1974 ( basic reference )

Abraham, Ralph., Jerrold E. Marsden and Tudor Ratiu : Manifolds, Tensor Analysis and Applications, Applied Mathematical Sciences, Vol. 75, New York, Springer-Verlag, 1998.

Bott, Raoul, R. Bott and Loring W.Tu. : Differential Forms in Algebraic Topology, Graduate Texts in Mathematics. Vol.82, Reprint edition. New York, Springer-Verlag, 1995.

For the examples we refer to the books of

Greenberg : Lectures on Algebraic Topology, W.A. Benjamin, 1967.

Munkres : Elements of Algebraic Topology, Addison-Wesley, 1984.

Hirsch : Differential Topology, Springer, 1976.

## **A – 2.3 : Introduction to Algorithms - II : 50 marks**

### **Theory - 35, Assignment - 15 (Computer lab access is necessary and mandatory)**

Advanced design and analysis techniques : Dynamic Programming - Matrix Chain Multiplication, Elements of dynamic programming, longest common subsequence, optimal polygon triangulation. Greedy Algorithms – Activity selection problem, Elements of greedy strategy, Huffman codes, Amortized analysis – The aggregate method, The accounting method, The potential method, Dynamic tables. 20 lectures

Graph problems: Graph searching - BFS, DFS, topological sort; connected and biconnected components; minimum spanning trees - Kruskal’s and Prim’s algorithms, Single-source shortest paths - Dijkstra's algorithm, Bellman-ford algorithm, All-pair shortest paths – Shortest paths and matrix multiplication, Floyd-Warshall algorithm, Maximum flow – Flow networks, Ford-Fulkerson method, Maximum bipartite matching. 14 Lectures

Algebraic problems: Evaluation of polynomials with or without preprocessing. Winograd’s and Strassen’s matrix multiplication algorithms and applications to related problems, FFT, simple lower bound results. 10 Lectures

String processing: String searching and Pattern matching, Knuth-Morris-Pratt algorithm and its analysis. 6 Lectures

Computational geometry : Line-segment properties, Determining whether any pair of segments intersects, Finding the convex hull, Finding the closest pair of points. 10 Lectures

NP-completeness: Turing machines, Church's Thesis, P and NP, NP-completeness, statement of Cook's theorem, some standard NP-complete problems, NP-hard problems, NP-hard problems, Graph Realization in two and three-dimensional real spaces.

20 Lectures

Approximation algorithms : Vertex covering, Traveling salesman problem, Set covering, Subset-sum problem.

10 Lectures

**Text books :**

1. T. H. Cormen, C.E. Leiserson and R.L.Rivest: Introduction to Algorithms, Prentice Hall of India, New Delhi, 1998.
2. Bernhard Korte and Jens Vygen : Combinatorial Optimization : Theory and Algorithms, Springer, 2005

**References:**

1. A. M. Tannenbaum and M. J. Augesstein: Data Structures Using PASCAL, Prentice Hall, New Jersey, 1981.
2. E. Horowitz and S. Sahni: Fundamentals of Data Structures, CBS, New Delhi, 1977.
3. A. Aho, J. Hopcroft, and J. Ullman: Data Structures and Algorithms, Addison-Wesley, Reading, Mass., 1983.

## **A – 2.4 : Fluid Mechanics II (50 marks)**

Basic thermodynamics of one compressible fluids:

Six governing equations of fluid motion, Crocco-vazsonyi equation. Propagation of small disturbances in a gas. Mach number. Dynamics similarity of two flows. Circulation theorem. Permanence of irrotational motion. Bernoulli's integral for steady isentropic and irrotational motion. Polytropic gas. Critical speed. Equation satisfied velocity potential and stream functions. Prandtl-Mayer fluid past a convex corner.

Steady flow through a De Laval nozzle. Normal and oblique shock wave shock polar diagram one dimensional similarity flow.

Steady linearised subsonic and supersonic flows. Prandtl-Glauert transformation. Flow along a wavy boundary flow past a slight corner. Jansen-Rayleigh method of approximation. Thin supersonic wind Ackeret's formula.

Legendre and Molenbroek transformations Chaplygin's equation for stream function. Solution of Chaplygin's equation. Subsonic gas jet problem limiting line. Motion due to a two dimensional source and a vortex Karman-Tsien approximation. Two dimensional steady flow : Riemann invariance. Method of characteristic. Transonic flow. Law transonic similarity. Euler's-Tricomi equation and its fundamental solution. Hypersonic flow.

### **Reference Books:**

1. Hydrodynamics –A.S.Ramsay(Bell)
2. Hydrodynamics – H. Lamb(Cambridge)
3. Fluid mechanics – L.D.Landou and E.M.Lifchiz(Pergamon),1959
4. Theoretical hydrodynamics –L.M.Thomson
5. Theoretical aerodynamics –I.M.Milne-Thomson;Macmillan, 1958
6. Introduction to the theory of compressible flow –Shih-I.Pai; Van Nostrand, 1959
7. Inviscid gas dynamics – P.Niyogi, Mcmillan, 1975(india)
8. Gas dynamics – K.Oswatitsch(english tr.) academic press, 1956

## A-2.5 Modelling of Biological Events II (50 marks)

### **Models for Molecular Events:**

Law of mass action, basic enzyme reaction, the quasi-steady-state assumption and its consequences, validity of the quasi-steady-state assumption, motion kinetics, cooperative kinetics, stability of activator –inhibitor and positive feedback systems.

### **The Chemostat Model:**

Bacterial growth in a chemostat, formulation of the chemostat model, stability analysis of the steady states, construction of phase-plane diagram, saturating nutrient consumption rate (Monod model).

### **Diffusion Model:**

The general balance law, Fick's law, diffusivity of motile bacteria.

### **Models for Developmental Pattern Formation:**

Background, model formulation, spatially homogeneous and inhomogeneous solutions, Turing model, conditions for diffusive stability and instability, pattern generation with single species model.

### **Models of Biological Oscillators:**

Oscillation in chemical systems, Goodwin's model, its stability and oscillations, simple two species oscillators- parameter domain determination for oscillations.

### **Models for Population Genetics:**

Introduction, basic model for inheritance of genetic characteristic, Hardy-Wienberg law, models for genetic improvement, selection and mutation- steady state solution and stability theory.

### **References:**

1. L.A.Segel (1984): Modelling Dynamical Phenomena in Molecular Biology, Cambridge University Press.
2. J.D.Murray (1990): Mathematical Biology, Springer and Verlag.
3. Leach Edelstein-Keshet (1987): Mathematical Models in Biology, The Random House/ Birkhauser Mathematics Series.

## A – 2.6 : Mathematical Theory of Elasticity II (50 Marks)

**Solution by means of functions of a complex variable :** Plane Stress and Plane Strain Problems. Solution of Plane Stress and Plane Strain Problems in Polar Co ordinates. General Solution for an infinite plate with a circular hole. An infinite Plate under the Action of Concentrated Forces and Moments.

**Three dimensional problems :** Beam Stretched by its own weight. Solution of differential equations of equilibrium in terms of stresses. Stress function. Reduction of Lamé and Beltrami equations to biharmonic equations. Relvin and Boussinesq-Papkovich solution. Pressure on the Surface of a Semi-infinite Body.

**Theory of thin plates :** Basic equations for bending of plates. Boundary conditions. Navier's and Levy solutions for rectangular plates. Circular Plate. Cylindrical Bending of Uniformly Loaded Plates.

**Variational methods :** Theorems of Minimum Potential Energy. Theorems of Minimum Supplementary Energy. Uniqueness of Solutions. Reciprocal theorem of Betti and Rayleigh – applications. Solution of Eulevs equation by Ritz, Galerkin and Rantorovich method.

Reference:

1. A Treatise on The Mathematical Theory of Elasticity – A. E. Love
2. Mathematical Theory of Elasticity - I. S. Sokolnikoff
3. Theory of Elasticity – S. Timoshenko and J. N. Goodier
4. Elasticity Theory and Applications – A. S. Saada
5. Foundations of Solid Mechanics – Y. C. Fung
6. Theory of Elasticity – Y. A. Amenzade
7. Applied Elasticity – Zhilun Xu
8. Wave Propagations in Elastic Solids – J. D. Achenbach
9. Elasto-dynamics – A. C. Eringen
10. Wave Motion in Elastic Solids – K. F. Graff
11. Applied Elastity – Chi-The Wang.

## **A – 2.7 : Principles of Operations Research II : 50 marks**

### **Sequencing**

Sequencing problems, Solution of sequencing problems, Processing  $n$  jobs through two machines, Processing  $n$  jobs through three machines, Optimal solutions, Processing of two jobs through  $m$  machines, Graphical method of solution, Processing  $n$  jobs through  $m$  machines.

### **Project Scheduling and Network Analysis**

Project scheduling by PERT and CPM, Construction of a network, Fulkerson's  $i, j$  rule, Errors and dummies in Nnetwork, Critical path analysis, Forward and backward pass methods, Floats of an activity, Project costs by CPM, Crashing of an activity, Crash-cost slope, Time-cost trade off, Solution of network problems using Simplex technique. Time estimates for PERT, Probability of completion of a project within a scheduled time.

### **Replacement Models**

Replacement problem, Types of replacement problems, Replacement of capital equipment that varies with time, Replacement policy for items where maintenance cost increases with time and money value is not considered, Money value, Present worth factor (pwf), Discount rate, Replacement policy for item whose maintenance cost increases with time and money value changes at a constant rate, Choice of best machine, Replacement of low cost items, Group replacement, Individual replacement policy, Mortality theorem, Recruitment and promotional problems.

### **Inventory Problems**

Introduction, Inventory problems, Inventory parameters, Variables in inventory problems, Controlled and uncontrolled variables, Classification of inventory models, Deterministic elementary inventory models, Economic lot size formula and its properties, Problems.

### **Reference Books**

1. Operations Research - S.D. Sharma
2. Operations Research - Kanti Swarup, P.K. Gupta and Manmohan
3. OR methods and Problems - Sasieni Maurice, Arther Yaspan, Lawrence Friedman
4. Operations Research - H.S. Taha

## **A – 2.8 : Quantum Mechanics (50 marks)**

**Scattering Theory:** Kinematics of the scattering Process: Differential and total cross section. Laboratory and centre of mass system. Wave Mechanical Picture of scattering; scattering amplitude, boundary conditions, Optical Theorem. Born Approximation, its validity. Partial wave analysis. Phase shifts.

**Relativistic Quantum Mechanics:** KLEIN-GORDON equation and its difficulties, Plane wave solutions, charge, current densities. Dirac equation, Plane wave solutions, dirac matrices, charge current conservation, Spin of Dirac particle. Significance of negative energy states, Concept of antiparticle, dirac hole theory. Non-relativistic correspondence of Dirac equation. Lorentz covariance of dirac equation. Parity, Charge conjugation, time reversal in Dirac equation.

### **References:**

1. I.Schiff: “Quantum Mechanics”.
2. P.M. Mathews and K Venkatesan: “Quantum Mechanics”.
3. P.A.M Dirac : “The Principles of Quantum Mechanics”.

## **Unit 4.4 and Unit 4.5 : Two courses from B-2.1 to B-2.39**

(\*corresponding to the course selected in Unit 3.4 and unit 3.5 in the 2<sup>nd</sup> yr 1<sup>st</sup> semester.  
If a student has studied B-1.4 and B-1.34 in the 2<sup>nd</sup> year 1st semester then the student has to go for B-2.4 and B-2.34 in this semester)

B - 2.1	Advanced Complex Analysis II
B - 2.2	Advanced Differential Geometry II
B - 2.3	Artificial Intelligence & Soft Computing II
B - 2.4	Astrophysics II
B - 2.5	Combinatorial Mathematics II
B - 2.6	Commutative Algebra and Algebraic Geometry II
B - 2.7.	Computational Biology II
B - 2.8	Computational Fluid Dynamics Practical
B - 2.9.	Computational Solid Mechanics II
B – 2.10	Image Processing
B – 2.11	Introduction to Computer Networks
B - 2.12	Coupled Fields of Solid Mechanics & Plasticity II
B - 2.13	Differential Geometry and its application II
B - 2.14	Dynamical Meteorology & Numerical weather prediction II
B - 2.15	Dynamical Oceanography II
B - 2.16.	Elastodynamics II
B - 2.17	General Theory of Relativity and Cosmology II
B - 2.18	Generalized function & Wavelet Theory II
B - 2.19	Graph Theory II
B - 2.20	Information Theory & Coding II
B - 2.21	Magneto Fluid Mechanics II
B - 2.22	Mathematical Ecology II
B - 2.23	Mathematical Statistics II
B - 2.24	Measure and Topology II
B - 2.25	Mechanics of Viscous Fluid and Boundary Layer Theory II
B - 2.26	Nonlinear and Dynamic Programming II
B - 2.27	Introduction to Cryptography
B - 2.28	Introduction to Compiler Design
B - 2.29	Operator Algebra II
B - 2.30	Operator Theory II
B - 2.31	Plasma Mechanics II
B - 2.32	Probability & Stochastic Processes II
B - 2.33	Production and Inventory Control II
B - 2.34	Quantum Field Theory & Statistical Mechanics II
B - 2.35.	Queuing Theory and Game Theory II
B - 2.36	Renewable Bio-economic Modelling and Epidemiology II
B - 2.37	Theory of Marketing Decisions II

- B - 2.38. Theory of Semi-groups II  
 B - 2.39 Topological Groups & Harmonic Analysis II

### **B – 2.1: Advanced Complex Analysis II (50 Marks)**

Harmonic Functions, Characterization of Harmonic Functions by Mean-Value Property, Poisson's Integral Formula, Dirichlet Problem for a Disc.

Doubly Periodic Functions, Weierstrass Elliptic Functions.

Meromorphic Functions, Expansions, Definition of the functions  $m(r,a)$ ,  $N(r,a)$  and  $T(r)$ . Nevanlinna's First Fundamental Theorem, Cartan's Identity and Convexity Theorems, Order of Growth, Order of a Meromorphic Function, Comparative Growth of  $\log M(r)$  and  $T(r)$ , Nevanlinna's Second Fundamental Theorem, Estimation of  $S(r)$  (statement only), Nevanlinna's Theory of Deficient Values, Upper Bound of the Sum of Deficiencies.

#### **References :**

- Conway, J.B., *Functions of one complex variable*, Second Edition, Narosa Publishing House.  
 Ahlfors, L.V., *Complex Analysis*, McGraw-Hill, 1979.  
 Rudin, W., *Real and Complex Analysis*, McGraw-Hill Book Co., 1966.  
 Hille, E., *Analytic Function Theory* (2 vols.), Gonn & Co., 1959.  
 Titchmarsh, E.C., *The Theory of Functions*, Oxford University Press, London.  
 Markusevich, A.I., *Theory of Functions of a Complex Variable*, Vol. I, II, III.  
 Copson, E.T., *An Introduction to the Theory of Functions of a Complex Variable*.  
 Hayman, W.K., *Meromorphic Functions*.  
 Kaplan, W., *An Introduction to Analytic Functions*.

### **B – 2.2 : Advanced Differential Geometry II (50 Marks)**

Principal fibre bundle, Linear frame bundle, Associated fibre bundle, Vector bundle, Tangent bundle, Induced bundle, Bundle homomorphisms.

Linear and affine connections, Riemannian connection, Riemannian manifolds, Curvature tensors, Sectional curvature, Schur's theorem,

Geodesics in a Riemannian manifold, Projective curvature tensor, Conformal curvature tensor, Submanifolds and hypersurfaces.

#### **References**

1. Foundation of differential Geometry (vol-1) :- S.KOBAYASHI and
2. K.NOMIZU.
3. An Introduction to Differentiable Manifolds and Riemannian
4. Geometry :- W.M.BOOTHBY.



5. Introduction to Differentiable Manifolds : - L.AUSLANDER and
6. R.E.MACKENZIE.
7. Lectures on Differential Geometry : - S.S.CHERN,W.H.CHEN and
8. K.S.LAM.

### **B – 2.3 : Artificial Intelligence and Soft Computing II : 50 marks**

Algebra on Fuzzy quantities. Logical aspects of fuzzy sets. Fuzzy relations.  
 Neural networks and learning processes. Single-layer perceptions. Multi-layer perceptions. Neuro-dynamics  
 Mathematical foundations of genetic algorithms. Operators and techniques in genetic search.

#### **References:**

1. Principles of Artificial Intelligence by Nils J. Nilsson – Springer.
2. Artificial Intelligence and Soft Computing by Amit Konar – CRC Press.
3. Neural Networks by Simon Haykin – Pearson
4. Genetic Algorithms by David E. Goldberg – Addison-Wesley

### **B – 2.4 : Astrophysics II (50 marks)**

4.
  - 1.1 Plasma, black Body, Cherenkov & Synchrotron Radiation
  - 1.2 Accretion as source of radiation
  - 1.3 Quasar as source of radiation, Compton effect effect
  - 1.4 Bremsstrahlung Radiation.
5. Formation of Galactic Structure – different Theories : -
  - 1.1 Formation of our Galaxy.
  - 1.2 Formation of Galaxy in Evolutionary Universe.
  - 1.3 Formation of Galaxy in Steady State Universe.
  - 1.4 Possibility of galactic structure formation through Explosion.
6.
  - 1.1 Hubble's Law & Expansion of Universe – Big Bang Model
  - 1.2 Uniformity of Large Scale Structure of the Universe .
  - 1.3 Origin of Cosmic Rays
  - 1.4 Origin of Galaxies and the Universe.

#### **References:**

1. The Structure of the Universe – J.V. narlikar
2. Astrophysics – B. Basu
3. Astrophysics – B. Basu
4. Astrophysical Concept – M. Harmitt
5. Galactic Structure – A. Blaauw & M. Schmidt

6. Large Scale Structure of Galaxies – W.B. Burton
7. The Milky Way – B.T. Bok & P.F. Bok.
8. Cosmic Electrodynamics – J.H. Piddington

### **B – 2.5 : Combinatorial Mathematics II (50 Marks)**

Steiner Triple Systems, Packing and covering.  
 Ramsey's Theorem, Pigeonhole Principle, Bounds for Ramsey Number.  
 Design, Fisher's Inequality.  
 Linear code, Error Correcting Code, Hamming Code.  
 Generating Function, Recurrence Relations, Solutions of Recurrence Relations by  
 Generating Functions method.

#### **References :**

1. COMBINATORICS – Topics, Techniques, Algorithms *by* Peter J. Cameron (Cambridge University Press).
2. A COURSE IN COMBINATORICS *by* J. H. Lint & R. M. Wilson (Cambridge University Press).
5. DISCRETE AND COMBINATORIAL MATHEMATICS *by* Ralph P. Grimaldi (AWL).
6. DISCRETE MATHEMATICS FOR COMPUTER SCIENTISTS & MATHEMATICIANS *by* Joe L. Mott, Abraham Kandel & Theodore P. Baker (Prentice-Hall).

### **B – 2.6 : Commutative Algebra and Algebraic Geometry II (50 Marks)**

#### **Commutative algebra :**

Valuation Rings, Discrete Valuation Rings, Dedekind Domains, Fractional and Invertible Ideals.

Topologies and Completions, Filtrations, Graded Rings and Modules, The Associated Graded Ring.

Hilbert Functions, Dimension Theory of Noetherian Local Rings, Hilbert-Samuel Polynomials, Krull's Principal Ideal Theorem, Regular Local Rings, Transcendental Dimension.

**Note :** This course is based on the books [1], [2]. See [4] for Schmidt and Lüroth Theorems and Elimination Theory.

#### **Algebraic Geometry :**

Affine Space, Algebraic Sets, The Ideal of a Set of Points, Irreducible Components of an Algebraic Set, Algebraic Subsets of the Plane, Hilbert Nullstellensatz.

Affine Varieties, Coordinate Rings, Polynomial Maps, Coordinate Changes, Rational Functions and Local Rings, Discrete Valuation Rings, Forms, Ideals with Finite Number of Zeros.

Affine Plane Curves, Multiple Points and Tangent Lines, Multiplicities and Local Rings,

Intersection Numbers.

Resultants and Discriminants of Polynomials, Introduction to Elimination Theory.

**Note :** This course is based on the book [3]; Chapters 1-3. See [4] for Elimination Theory.

**References :**

- Atiyah, M., MacDonald, I.G., *Introduction to Commutative Algebra*, Addison-Wesley, 1969.
- Gopalakrishnan, N.S., *Commutative Algebra*, Oxonian Press Pvt. Ltd., New Delhi, 1988.
- Fulton, W., *Algebraic Curves*, W.A. Benjamin, INC., 1969.
- Musili, C.S., *Algebraic Geometry for Beginners*, TRIM 20, Hindustan Book Agency, New Delhi, 2001.
- Walker, R., *Algebraic Curves*, Dover, NY, 1962.
- Dummit, D.S., Foote, R.M., *Abstract Algebra*, Second Edition, John Wiley & Sons, Inc., 1999.
- Lang, S., *Algebra*, Addison-Wesley, 1993.

**B – 2.7 : Computational Biology II (50 marks)**

**5. Constructing Phylogenetic Trees.**

Phylogenetic Trees. Tree Construction: Distance Methods – Basics. Tree Construction: Distance Methods – Neighbour Joining. Tree Construction : Maximum Parsimony.

**6. Genetics.**

Mendelian Genetics. Probability Distributions in Genetics. Linkage. Gene Frequency in Populations.

**7. Infectious Disease Modelling.**

Elementary Epidemic Models. Threshold Values and Critical Parameters. Variations on a Theme.. Multiple Populations and Differentiated Infectivity.

**8. Curve fitting and Biological Modelling.**

Fitting Curves to Data. The Method of Least Squares. Polynomial Curve Fitting.

**References:**

1. Elizabeth A. Allman and John A. Rhodes: *Mathematical Models in Biology: An Introduction*. Cambridge University Press (2004)

## **B –2. 8 : Computational Fluid Dynamics (Practical) (50 marks)**

Solution of one-dimensional heat conduction equation using Explicit and Implicit scheme. Solution of 2-D heat conduction equation using ADI method. Solution of partial differential equation using Lax-Wendroff and Mac Cormack's scheme. Solution of Navier-Stokes equation using stream function vorticity formulation. Solution of some problems on fluid dynamics using MAC method and SIMPLE algorithm

*Prerequisites* : Programming in C

### **References:**

8. P. Niyogi, S. K. Chakraborty and M. K. Laha- Introduction to computational fluid Dynamics, Pearson Education, Delhi 2005.
9. C. A. J. Fletcher- Computational Techniques for Fluid Dynamics, Vol-I and Vol-II, Springer, 1988.
10. R. Peyret and T. D. Taylor- Computational Methods for Fluid Flow, Springer 1983.
11. J. F. Thompson, Z.U.A Warsi and C. W. Martin, Numerical Grid Generation, Foundations and Applications, North Hollamm, 1985.
12. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Trans., Pergamon Press, 1989.
13. H. Schlichting and K. Gersten- Boundary Layer Theory, 8<sup>th</sup> Ed., Springer 2000.

## **B –2. 9 : Computational Solid Mechanics II (Practical) :**

Finite Element Method : Computer programming in C or C++ using Triangular elements. Plotting of Mesh points. Solving problems in Elasticity.

Boundary Element Method : Numerical computation of some potential and elasticity problems using constant, linear and quadratic elements.

### **References :**

1. Finite Element Method : J.N.Reddy.
- 2..Finite Element Procedures : K.J.Bathe
- 3.Introduction to Finite and Boundary Element Method : G.Beer and J.O.Watson.
- 4.The Finite Element Method – Vol.I (The Basis): O.C.Zienkiewicz and R.L.Taylor
- 5.The Finite Element Method – Vol. II (Solid Mechanics) : O.C.Zienkiewicz and R.L.Taylor.
6. Programming the Finite Element Method : I.M. Smith, D.V.Griffiths.
7. Boundary Elements- An Introductory Course : C.A.Brebbia and J.Dominguez.
8. An Introduction to Boundary Element Methods : P.K. Kathy.
9. The Boundary Element Method in Engineering ; A.A.Becker.

10. Computational Elasticity : M. Ameen.
11. Boundary Element Techniques : C.A.Brebbia, J.C.F.Tellers, L.C.Wrobel
12. Underlying Principles of Boundary Element Method : D.J.Cartwright.
13. Programming the Boundary Element Method – An introduction for Engineers : G. Beer.

### **B – 2.10. Image Processing (50 Marks)**

**Theory - 30, Assignment - 20**      *(Computer lab access is necessary and mandatory)*

Introduction, image definition and its representation, neighborhood metrics, image processing systems, 2-D orthogonal transformations of images (DFT, DCT, HT, KLT), enhancement, contrast stretching, histogram specification, local contrast enhancement, smoothing and sharpening, spatial/ frequency domain filtering, segmentation, pixel classification, grey level thresh-holding, global/local thresh-holding, edge detection operators, region growing, split/merge techniques, image feature/primitive extraction, line detection, border following, Hough transform, medial axis transform, skeletonization / thinning, shape properties, compression, Huffman coding, block truncation coding, run-length coding, some applications.

**References:**

1. R. C. Gonzalez and R. E. Woods: Digital Image Processing, Addison-Wesley, California, 1993.
2. A. Jain: Fundamentals of Digital Image Processing, Prentice Hall of India, New Delhi, 1989.
3. B. Chanda and D. Dutta Majumder: Digital Image Processing and Analysis, Prentice Hall of India, New Delhi, 2000.

### **B – 2.11: Introduction to Computer Networks (50 Marks)**

**Theory - 30, Assignment - 20**      *(Computer lab access is necessary and mandatory)*

Introduction: Computer networks and distributed systems, classifications of computer networks, layered network structures. 5 lectures

Data Communication Fundamentals: Channel characteristics, various transmission media, different modulation techniques. 7 lectures

Network Structure: Concepts of subnets, backbone and local access; Channel sharing techniques-FDM, TDMj Polling and concentration, message transport: circuit, message and packet-switching, topological design of a network. 12 lectures

Data Link Layers: Services and design issues, framing techniques, error handling and flow control, stop and wait, sliding window and APRANET protocols, HDCLC standard. 10 lectures

Network Layer: Design issues, internal organization of a subnet, routing and congestion control techniques, network architecture and protocols, concepts in protocol design, CCITT recommendation X.25 12 lectures

LANs and their Interconnection: Basic concepts, architectures, protocols, management and performance of Ethernet, token ring and token bus LANS; Repeaters and Bridges. 10 lectures

Internet: IP protocol, Internet control protocols - ICMP, APR and RAPP, Internet routing protocols OSPF, BGP and CIDR. 12 lectures

ATM: ATM switches and AAL layer protocols. 10 lectures

Network Security: Electronic mail, directory services and network management. 10 lectures

Wireless and mobile communication: Wireless transmission, cellular radio, personal communication service, wireless protocol. Network planning, Gigabit and Terabit technology, CDMA, WCDMA, WDM, optical communication networks. 12 lectures

### **References:**

1. A. Tannenbaum: Computer Networks, 3rd ed., Prentice Hall India, 1996.
2. W. Stallings: ISDN and Broadband ISDN With Frame Relay and ATM, Prentice Hall, Englewood Cliffs, 1995.

## **B – 2.12 : Coupled fields of Solid Mechanics and Plasticity II (50 marks)**

### **Thermoelasticity : 25 Marks**

Fundamental relations and equations of thermoelasticity . First and second law of thermodynamics. Principle of conservation of energy and entropy balance . The constitutive relations of thermoelasticity. Duhamel – Neumann relations . Generalized heat conduction equation. Theorem of conservation of energy and the uniqueness theorem of thermoelasticity. Compatibility equations. Reciprocity theorem. Simple problems of thermoelasticity concerning disk, cylinder and sphere. Thermoelastic complex potentials. Time harmonic plane waves, transverse, longitudinal and transient waves in coupled thermoelasticity . Generalized thermoelasticity : Equations of extended and temperature rate dependent thermoelasticity, Simple problems.

1. Thermal stress : N. Noda, R.B. Hetnarki and Y. Tanigawa.

2. Theory of thermal stress : B.A. Boley and J.H. Weiner.
3. Thermoelasticity : H. Purkus.
4. Dynamic problems of Thermoelasticities . W. Nowacki.

### **Viscoelasticity : 25 Marks**

Properties of viscoelastic materials : The Voigt model, the creep function, the Boltzmann superposition principle. The Maxwell model and the standard linear model. The relaxation function . The mechanical impedance function . Voigt model to creep function. Maxwell model to operator equation. Viscoelastic representations for pressure- volume effects. Stress-strain relations in different equations form. Boundary value problems. Reciprocity relations. Viscoelastic waves in an infinite medium . Time harmonic longitudinal, transverse and transient waves.

1. Foundations of solid mechanics : Y.C. Fung.
2. Wave Propagation in Elastic solid : J.D. Achenbach
3. Rheology Vol. I. Edited by Frederick R. Eirich.

### **B – 2.13 : Differential Geometry and its Applications II (50 marks)**

Introduction to special theory of relativity and general theory of relativity, Manifolds of special and general theories of relativity, Metric in a gravitational field, Motion of a free particle in a gravitational field, Einstein law of gravitation, Metrics with spherical symmetry, Killing equations and conservation law, Spinor formalism, General theory of relativity in spinor formalism.

References :

1. K.YANO, K.M.KON–Differentiable Manifold.
2. D.E.BLAIR–Contact Manifolds in Riemannian Geometry, Lecture Notes in Maths.
3. R.RESNIK–Introduction to Special Theory of Relativity.
4. D.S.LONDEN–An Introduction to Tensor Calculus, Relativity and Cosmology.
5. A.N.MATREEV–Mechanics and Theory of Relativity.

## **B – 2.14 : Dynamic Meteorology and NWP II (50 marks)**

11 : Unbalanced flow

Introduction, geostrophic adjustment, example of the Lagrangian method, the case of the anticyclone, divergence of parcels in a fluid, stream lines, the stream function.

12 : Euler and Lagrange

Introduction, geostrophic adjustment, example of the Lagrangian method, the case of the anticyclone, divergence of parcels in a fluid, stream lines, the stream function.

13 : Velocity

Introduction, circulation, vorticity, derivation of expressions for vorticity, relative and absolute vorticity, divergence-vorticity relation. A simple wave pattern, constant absolute vorticity trajectories, potential vorticity.

14 : The upper air synoptic chart

Introduction, pressure as vertical coordinate, Thermal wind Barotropic and baroclinic structure, vorticity on isobaric surface, velocity potential.

15 : Friction in the boundary layer of the atmosphere

Introduction, the Guldberg-Mohn approximation, Balanced frictional flow, The Newtonian concept of friction, The surface layer, The spiral or Ekman layer.

16 : Some more advanced equation

The divergence equation, The balanced equation, The omega equation.

17 : The Tropical cyclone

Introduction, structure and energy source, Genesis, Steering and development, movement, Development, Forecasting skill, problems.

18 : Synoptic observation and Numerical model :

Synoptic observation objective analysis, subjective analysis, stream lines, common synoptic patterns, weather associated with synoptic system, Tropical cyclone, genesis, steering, development, movement and forecasting skill, Numerical weather models.

Reference :

5. Dynamic Meteorology- A basic course : Adrian Gordon, Warwick Grace, Peter Schwerdtfeger, Ronald Byron-Scott.
6. Dynamical and Physical Meteorology : George J Haltiner, Frank Martin.
7. Introduction to Dynamical Meteorology : Holton.
8. Fundamental of atmospheric physics : Murray L Salby.



## **B – 2.15 : Dynamical Oceanography II (50 marks)**

### **Forced Motion**

Forced Motion due to Surface Stress: Ekman Transport. Ekman Pumping. Laminar Ekman Layer. Tide producing Forces. Barotropic Motion in the Sea: Forced Shallow Water Equation.

### **Baroclinic Response of the Ocean.**

Response of the Ocean to a Moving Storm. Spin Down by Bottom Friction. Buoyancy Forcing. Response to Stationary Forcing: Barotropic Case. Forced Baroclinic Vortex.

### **Effects of Slide Boundary**

Effects of Rotation on Selches and Tides in Narrow Channels and Gulfs. Poincare Waves in a Uniform Channel. Kelvin Waves. Modes in an Infinite Channel of Uniform Width. End Effects for Selches and Tides in a Gulf that is not Narrow. Adjustment to Equilibrium to a Channel. Tides. Storm Surges on an Open Coast Line. Forced Kelvin Waves. Coastal Upwelling. Continental Shelf waves. Coastally Trapped Waves. Eastern Boundary Currents.

### **Tropics**

Effects of Earth's Curvature: Shallow Water Equation on the Sphere. Potential Vorticity for a Shallow Homogeneous Layer. Equatorial Beta Plane. Equatorial Kelvin Wave. Equatorial Waveguide: Gravity Waves. Planetary Waves and Quasi-geostrophic Motion. Baroclinic Motion near Equator. Vertically Propagating Equatorial Waves. Adjustment under Gravity near the Equator. Transient Forced Motion. Potential Vorticity for Baroclinic Motion. Steady Forced Motion. Tropical Circulation of the Atmosphere. Tropical Ocean Currents.

### **Mid Latitude**

Mid Latitude Beta Plane. Planetary Waves. Spin up of the Ocean by an applied Wind Stress. Steady Ocean Circulation. Western Boundary Currents. Non-linear Quasi-geostrophic Flow. Small Disturbances on a Zonal Flow varying with Latitude and Height. Deduction of Vertical Motion from Quasi-geostrophic Equation.

### **Instabilities, Fronts and General Circulation**

Free Waves in Presence of Horizontal Temperature Gradient. Baroclinic Instability: the Eddy Problem. Baroclinic Instability: the Charney Problem. Necessary Condition for Instability. Barotropic Instability. Eddies in the Ocean. Fronts. Life-cycle of a Baroclinic Disturbance. General Circulation of Atmosphere.

### **References:**

5. O. M. Philips: Dynamics of the Upper Ocean. Cambridge University Press (1966)

6. J. Pedlosky: Geophysical Fluid Dynamics. Springer (1987)
7. G. P. Pickand: Descriptive Physical Oceanography. Oxford Pergaman Press (1975)
8. A. E. Gill: Atmospheric Ocean Dynamics. Academic Press (1982)

## **B – 2.16 : Elastodynamics – II (50 Marks)**

Two dimensional wave propagation. Plane elastic waves in a half-space with free boundary. Reflection of plane waves at a plane surface. Time harmonic SH-waves due to a line source on the free surface of a semi-infinite media. Moving line load on the surface of a half-space.

Elastic waves in infinite plates. Longitudinal waves in an infinite rod of circular cross section. Scattering of waves by cylindrical cavity. Scattered energy.

The Sommerfield diffraction problem. Diffraction of elastic waves by a crack. Diffraction of elastic waves by a rigid strip.

Nonlinear wave propagation : One dimensional nonlinear waves. Hyperbolic systems and characteristics. Formation and propagation of Shock waves.

### **References :**

1. Elastodynamics, Vol.-II : A.C.Eringen and E.S.Suhubi.
2. Wave Motion in Elastic Solids : K.F.Graff.
3. Stress waves in Solids : H.Kolsky.
4. Wave Propagation in Elastic Solids : J.D.Achenbach.
5. Wave Motion : J.Billingham and A.C.King.
6. Elastic Wave Propagation : A.Bedford and D.S.Drumheller.
7. Elastic Waves in Layered Media : W.M.Ewing, W.S.Jardetzky and F.Press.
8. On Wave propagation in Elastic Solids with cracks : Ch. Zhang and D. Gross.
9. Linear and Nonlinear Waves : G.B

## **B – 2.17 : General Theory of Relativity and Cosmology II (50 Marks)**

What is cosmology ? Homogeneity and isotropy of the universe. The Weyl Postulate. The cosmological principle. General relativistic cosmological models. Cosmological observations. The Olbers Paradox. The Friedman Cosmological Models (dust and radiation models). Cosmologies with a non-zero  $\lambda$  . Hubble's Law, the age of the Universe. Gravitational red shift and Cosmological redshift.

The spherically symmetric space-time : Schwarzschild solution. Particle orbits in the Schwarzschild space-time. Newtonian approximation. Photon orbits. Birkhoff's theorem. Equilibrium of Massive spherical objects. The Schwarzschild Interior solution. The interior structure of the star. Realistic stars and gravitational collapse. White dwarfs, Neutron stars.

Gravitational collapse of a homogeneous dust ball. Schwarzschild black hole . Simple idea of black hole physics.

### **Reference:s :**

1. General Relativity and Cosmology – J.V. Narlikar.
2. A first course in general relativity – B.F. Schutz.
3. Introduction to cosmology - J.V. Narlikar.
4. An Introduction to Mathematical Cosmology – J.N. Islam (Camb.Univ.Press).
5. Gravitation and Cosmology – S. Weinberg (J. Wiley and Sons.)
6. General Relativity, Astrophysics and Cosmology – Raychaudhuri, Banerji and Banerjee (Springer-Verlag).
7. Introduction to Cosmology – M. Ross (J. Wiley and Sons).

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## **B – 2.18 : Generalised Functions and Wavelet Theory II (50 Marks)**

### **Wavelets 50 Marks**

Introduction – An overview. From Fourier analysis to Wavelet analysis. Classification of Wavelets. Different ways of constructing wavelets. Orthonormal bases generated by single function. Wavelet frames for  $L^2(\mathbb{R})$ . Local sine and cosine bases and the

construction of some wavelets. The unitary operators and the smooth projections. Multiresolution analysis and construction of wavelets. Construction of compactly supported wavelets and estimates for its smoothness. Orthogonal wavelet packets. Band limited wavelets. Orthogonality. Completeness. Franklin wavelets and spline wavelets on the real line.

Applications: Wavelet data compression – Image compression, Transform image compression system, Wavelet image compression, Embedded coding and the wavelet-difference-reduction compression algorithm. Multiresolution audio compression. Denoising algorithms.

#### References :

1. An Introduction to Wavelets : C.K.Chui.
2. A first course on Wavelets : E.Hernandez and G.Weiss.
3. Wavelet Analysis : Howard L.Resnikoff and Raymond O. Wells Jr.
4. Wavelets : Mathematics and Applications : Edited by J.J.Benedetto and M.W.Frazier.
5. A primer on Wavelets and their scientific applications : J.S.Walker.
6. An Introduction to wavelets through linear algebra : Michael W. Frazier.
7. Wavelet Transform and Time-Frequency signal Analysis : L. Debnath.

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#### **B –2. 19 : Graph Theory II ( 50 Marks )**

**Planarity of Graphs** : Drawing graphs in a plane, Planar Graphs, Planar embeddings, Dual Graphs, Euler's Formula, Maximal Planar Graphs. Subdivisions, Kuratowski's Theorem, Convex Embedding. Planarity Testing Algorithm. Coloring of planar graphs, Edge-contraction, Five color theorem, Kempe's chain, Four Color Theorem (Statement only). Crossing Number. ( 10 )

**Directed Graphs** : Definitions and examples, Vertex degrees, Eulerian Digraphs, Orientations and Tournaments, Network and Flow problem, Max Flow – Min Cut Theorem, Algorithm for finding maximum flow. ( 10 )

**Matching** : Maximum Matching Problem, Hall's Marriage Theorem, Minimum covering problems : Vertex Cover, Konig-Egervary Theorem, Edge Cover and its characterization in terms of independence number. ( 10 )

**Practical** : Programs and Algorithms on problems of graph theory (mentioned above).

( 20 )

#### References :

1. Introduction to Graph Theory, Douglas B. West, Prentice-Hall of India Pvt. Ltd., New Delhi 2003.
2. Graph Theory, F. Harary, Addison-Wesley, 1969.

3. Basic Graph Theory, K.R. Parthasarathi, Tata McGraw-Hill Publ. Co. Ltd., New Delhi, 1994.
4. Graph Theory Applications, L.R. Foulds, Narosa Publishing House, New Delhi, 1993.
5. Graph Theory with Applications, J.A. Bondy and U.S.R. Murty, Elsevier science, 1976.
6. Graphs and Digraphs, G. Chartrand and L. Lesniak, Chapman & Hall, 1996.
7. Theory of Graphs, O. Ore, AMS Colloq. 38, Amer.Math.Soc., 1962.
8. Graph Theory, R. Gould, Benjamin / Cummings, 1988.
9. Graph Theory, J. Gross and J. Yellen, CRC Press, 1999.
10. Graph Theory with Applications to Engineering and Computer Science, Narsingh Deo, Prentice-Hall of India Pvt.Ltd., New Delhi, 1997.

### **B – 2.20: Information Theory and Coding II (50 Marks)**

The communication channel. Coding problems. Types of codes. Weight and distance. Block codes. Error detecting and error correcting codes. Linear codes. Hamming codes. Error-correction capabilities of linear codes. Hoffman codes. Generating matrices and Encoding. Parity-check matrices. Equivalent codes. Distance of a linear code. Maximum Likelihood Decoding (MLD) for linear codes. Dual codes Galay codes.

Cyclic linear codes : Polynomials and words. The cyclic codes. Polynomial Encoding and Decoding. Construction of cyclic codes. Dual cyclic codes.

BCH Codes : Finite fields. Minimal polynomials. Cyclic Hamming codes. BCH codes. Decoding 2 error-correcting BCH code.

#### **References :**

1. Elements of Information Theory : Thomas M. Cover and Joy A. Thomas.
2. The Theory of Information and coding : R.J. McEliece.
3. Information Theory, Inference and learning algorithms : David J.C. Kackay.
4. Information Theory : R. Ash.
5. An introduction to information theory : F.M. Reza.
6. Introduction to coding theory : J.H. Van Lint.
7. Coding Theory – The Essentials : D.G. Hoffman, D.A. Leonard, C.C. Lindner, K.T. Phelps, C.A. Rodger and J.R. Wall.
8. A first course in coding theory : R. Hill.
9. Error correcting coding theory : M.Y. Rhee.
10. Algebraic coding theory : E.R. Berlekamp.

### **B – 2.21 : Magneto Fluid Mechanics II (50 Marks)**

#### **MHD waves**

Alfven's wave and velocity, governing equation for Alfven's wave, MHD waves in compressible fluids (i.e. Magneto acoustic wave), reflection and refraction of Alfven's wave, Waves of finite amplitude, Dissipation effects in a viscous medium.

#### **Hydromagnetic stability**

Preliminaries, the method of small oscillations (the stability problem of an unbounded gravitating adiabatic gas in the presence of a magnetic field), the energy principle, the virial theorem, marginal stability, the Benard problem with a magnetic field.

### **Hydromagnetic shock waves**

Introduction, Stationary plane shock waves in the absence of a magnetic field, Stationary plane shock wave in the presence of a magnetic field normal to the direction of flow, oblique shocks.

### **Turbulence**

Introduction, spectral analysis, Hydromagnetic turbulence, Inhibition of turbulence by a magnetic field.

### **References:**

- 1) An introduction to Magneto-Fluid Mechanics-V, C.A>Ferraro & C. Plumpton.
- 2) Magneto Hydrodynamics – T.G. Cowling.
- 3) Text Book on Fluid Dynamics – F. Chorlton.
- 4) Magneto-Fluid Dynamics for Engineers and Physicists – K.R. Cramer & S.I. Pai.
- 5) Magneto Gas dynamics & plasma dynamics – S.I. Pai.

## **B – 2.22 : Mathematical Ecology II (50 Marks)**

### **Effect of Nutrients on autotrophy-herbivore interaction:**

Introduction, Models on nutrient recycling and its stability, Effect of nutrients on autotrophy herbivore stability, Models on herbivore nutrient recycling on autotrophic production.

### **Interactions of detritus and decomposers:**

Introduction, influence of decomposers on nutrient recycling. Donor-Controlled models and its stability, Effects of higher trophic levels in the detritus-based food chain.

### **Interaction of Ratio-dependent models:**

Introduction, May's model, ratio-dependent models of two interacting species, Two prey-one predator system with ratio-dependent predator influence- its stability and persistence.

### **Ecological Succession:**

Introduction, Succession in a three species food chain model, Models of succession under gradual changes in nutrient level in a food web.

### **Dynamics of Phytoplankton-Zooplankton system:**

Introduction, Models on phytoplankton-zooplankton system and its stability, Bio-control in plankton models with nutrient recycling.

### **References:**

4. H. I. Freedman - Deterministic Mathematical Models in Population Ecology
5. Mark Kot (2001): Elements of Mathematical Ecology, Cambridge Univ. Press.
6. D. Alstod (2001): Basic Populas Models of Ecology, Prentice Hall, Inc., NJ.

Home

## **B – 2.23: Mathematical Statistics II (50 Marks)**

Bayesian inference. Loss function. Decision function. Complete and minimal complete classes. Admissibility. Minimax solution. Bayes solution.

Inference related to linear models – one and several parametric functions. Setup with restrictions. Analysis of variance. One-way classified data. Two-way classified data with single/multiple observations per cell.

General theory of regression. Test for an assigned regression function.

Principal component analysis. Basic concepts of factor analysis.

Non-parametric inference. Distribution of order statistics. Robustness. Distribution-free methods. Standard non-parametric tests for location and independence.

Sequential probability ratio test and its properties. Fundamental identity. Sequential estimation.

### **References:**

4. Rao, C.R. – Linear Statistical Inference and its Applications.
5. Wilks, S.S. – Mathematical Statistics.
6. Ferguson, T.S. – Mathematical Statistics.

## **B -2. 24 : Measure and Topology II (50 Marks)**

### **Measure and Category :**

Measure and Category on Real Line, Countable and Uncountable Sets, Sets of First Category, Null Sets, Theorems of Cantor, Baire and Borel, Application of Category Method to Nowhere Differentiable Functions, Liouville Numbers, Algebraic and Transcendental Numbers, Measure and Category of the Set of Liouville Numbers. Lebesgue Measure in  $r$  space: Definitions and Principal Properties, Measurable Sets, The Lebesgue Density Theorem.

The Property of Baire, Its Analogy to Measurability, Properties of Regular Open Sets, Existence of Certain Non-Measurable Sets, Vitali Sets, Bernstein Sets, Ulam's Theorem, Kuratowski-Ulam Theorem.

Theorems of Lusin and Egoroff on Continuity of Measurable Functions and of Functions having the Property of Baire, Uniform Convergence on Subsets.

The Space of Automorphisms of an Interval, Effect of Monotone Substitution on Riemann Integrability, Nullsets Equivalent to Sets of First Category, Open Sets of First Category or Measure Zero, Montgomery's Lemma, Banach Category Theorem, Theorems of Marczewski and Sikorski, Cardinals of Measure Zero, Decomposition into a Nullset and a Set of First Category.

Similarities between the Classes of Sets of Measure Zero and of First Category, The Principle of Duality, Sierpinski-Erdos Duality Theorem, Examples of Duality, Lusin Sets and their Duals, Extended Principle of Duality.

Category Measure Spaces: Spaces in which Category and Measure Agree, Topologies generated by Lower Densities, The Lebesgue Density Topology.

## References :

1. Halmos, P.R., *Measure Theory*, Van Nostrand, Princeton, 1950.
2. Oxtoby, J.C., *Measure and Category*, Springer Verlag, 1980.
3. Berberian, S.K., *Measure and Integration*, Chelsea Publishing Company, NY, 1965.
4. Barra, G.de, *Measure Theory and Integration*, Wiley Eastern Ltd., 1981.
5. Rana, I.K., *An Introduction to Measure and Integration*, Narosa Publishing House, Delhi, 1997
6. Bartle, R.G., *The Elements of Integration*, John Wiley and Sons, Inc., NY, 1966.

## **B – 2.25 : Mechanics of Viscous fluids and Boundary layer Theory II (50 Maks)**

Fundamental concept of boundary layer when the Reynolds number is moderately large. Prandtl's equation of the boundary layer. Expressions of displacement thickness and momentum thickness of the boundary layer. Vorticity and stress components within the boundary layer in two dimensional motion. Separation of boundary layer from an obstacle.

Blasius equation for steady two dimensional motion past a flat plate and its solution in the form of an infinite series. Boundary layer for two dimensional steady converging radial flow between two non parallel walls. Boundary layer for two dimensional jet. Flow symmetrical about a free stream lines. Problem of steady three dimensional jet. Karman's integral equation of the boundary layer ; interpretation of its terms. Alternative form of integral equation in term of displacement, thickness and momentum thickness. Application of Karman's integral equation in the study of the approximate solutions of steady two dimensional flow past a flat plate and comparison with the corresponding exact solutions; calculations of frictional resistance on both sides of the plate and checking of errors. Application of this method by assuming liner, quadratic, cubic, and biquadratic distribution of velocity. Lamb's Trigonometric solution. Mises' Transformation of boundary layer equation into an equation of the conduction of heat with variable coefficient of conduction.

Non steady boundary layers, method of successive approximation and its application in the case of a flat plate impulsively set in motion. Unsteady motion of oscillatory cylinder and deduction of oscillatory motion of a piston.

## References :

1. Viscous flow theory, Vol.I --- S.I. Pai
2. Hydrodynamics --- H. Lamb
9. New methods in laminar boundary layer theory --- D. Meksyn.
10. Elementary treatise on hydrodynamics and sound --- A.B. Besset.
11. Modern developments in fluid dynamics --- S. Goldstein.
12. Boundary layer theory --- H. Schlichting.
13. Laminar boundary layers --- L. Rosenhead.



## **B – 2.26 : Non-linear and Dynamic Programming II (50 Marks)**

Multi-stage process of discrete and deterministic type. Associated functions. Recurrence relations. Continuous Multi-stage process. Trajectory process. Trajectory in non-homogenous atmosphere. Rabbit and dog problem.

Multi-stage decision process. of discrete and deterministic type. Return function policy. Bellman's optimality principle. Recurrence relation for return function. Continuous Multi-stage decision process. Calculus of variations as special case.

Deduction of Eikonal equation by the principle of Multi-stage decision process. Shortest path in a network. Discrete and continuous feedback control process. Solution of some non-linear programming problems by dynamic programming.

Some existence theorems for fundamental functional equations in dynamic programming.

### **References:**

1. Avriel – Nonlinear Programming.
2. Mangasarian – Nonlinear Programming
3. Kambo – Mathematical Programming Techniques.
4. Bellman and Kalaba – Nonlinear Programming and Modern Control Theory.

## **B -2. 27 : Introduction to Cryptography : 50 Marks**

Public Key Cryptography, RSA Cryptosystem, Diffie-Hellman and the Digital Signature Algorithm, Secret Sharing, Coin Flipping, Passwords, Signatures, and Ciphers, Practical Cryptosystems and Useful Impractical Ones.

Complexity of Computations, Big- $O$  Notation, Length of Numbers, Time Estimates, P, NP, and NP-Completeness.

Hidden Monomial Cryptosystems: The Imai-Matsumoto System, Patarin's Little Dragon, Systems That Might Be More Secure.

Combinatorial-Algebraic Cryptosystems: Irrelevance of Bassard's Theorem, Concrete Combinatorial-Algebraic System, The Basic Computational Algebra Problem, Cryptographic Version of Ideal Membership, Linear Algebra Attacks, Designing a Secure System.

**Note :** This course is based on the book [5]; Chapters 1-5.

### References :

1. Ireland & Rosen, *A Classical Introduction to Modern Number Theory*, Springer.
2. Niven, I., Zuckerman, S.H., Montgomery, L.H., *An Introduction to the Theory of Numbers*, Wiley.
3. Serre, J.-P., *A Course in Arithmetic*, Springer.
4. Cassels, J.W.S., Frolich, A., *Algebraic Number Theory*, Cambridge.
5. Koblitz, N., *Algebraic Aspects of Cryptography*, Springer.
6. Delfs, H., Knebl, H., *Introduction to Cryptography*, Springer, 2003.

## **B -2. 28. Introduction to Compiler Design (50 Marks)**

**Theory - 30, Assignment - 20** (Computer lab access is necessary and mandatory)

Introduction: Compiler, phases and passes, bootstrapping, finite state machines and regular expressions and their applications to lexical analysis, implementation to lexical analysers, lexical-analyser generator; LEX-compiler. 15 lectures

Syntax Analysis: Formal grammars, and their application to syntax analysis, BNF notation, ambiguity, LL(k) and LR(k) grammar, bottom-up and top-down parsers, operator precedence, simple precedence, recursive descent and predictive parsers, LR(k) parsers, parse table generation, grammars in YACC. 25 lectures

Syntax directed translation: Quadruples, triples, 3-address code, code generation for standard constructs with top-down and bottom-up parsers, procedure calls, record structuring. 15 lectures

Code optimization: Loop optimization, DAG analysis, loop identification by flow dominance, depth-first search, reducible flow graphs, legal code motion, induction variables, data flow analysis, u-d and d-u chains, copy propagation, elimination of global sub-expressions, constant folding, code hoisting, forward and backward data flow equations, inter procedural data flow analysis. 15 lectures

Code generation: Problems in code generation, code generator, register assignment and allocation problems, usage count, code generation from DAG, peephole optimization. 10 lectures

Symbol table: Data structure and management, runtime storage administration, error detection and recovery; Lexical, syntactic and semantic errors, case studies with real life compilers. 10 lectures

**References:**

1. A. V. Aho, R. Sethi and J. Ullman: Compilers: Principles, Techniques and Tools, Addison- Wesley, California, 1986.
2. A. Appel: Modern Compiler Implementation in Java, Cambridge Univ. Press, London, 1997.
3. J. R. Levine, T. Mason and D. Brown: lex & yacc, SPD, O'REILLY, Calcutta, 1999.

**B –2. 29 : Operator Algebra II (50 Marks)**

**Von Neumann Algebras :**

Von Neumann algebras, Monotone sequence of operators, Range Projections, The Commutant, The Double Commutant theorem, The Kaplansky Density theorem,  $L^\infty$  as Von Neumann Algebra, Maximal Abelian Algebras, Abelian Von Neumann Algebras, Cyclic and separating vectors, Representation of Abelian Von Neumann Algebras, The  $L^\infty$  functional calculus, Connectedness of the unitary group, The projection lattice, Kaplansky's formula, The centre of a Von Neumann Algebra, Various types of projections.

**References :**

1. Bonsall and Duncan, Complete Normed algebras, Springer-Verlag.
6. Kadison and Ringrose, Fundamentals of operator theory, Vol. I and II, Academic press.
7. Rickart, General theory of Banach Algebras, : Van Nostrand.
8. W. Arveson, An invitation to C\*-Algebras, Springer-Verlag.
9. Palmer, Banach Algebras and the general theory of C\*-algebras, Cambridge University Press.

**B – 2.30: Operator Theory II (50 Marks)**

**Selfadjoint operators: (50 Marks)**

Spectral properties of bounded selfadjoint linear operators on a complex Hilbert space, Positive operators, Square root of a positive operator, Projection operators, Spectral family of a bounded selfadjoint linear operator and its properties, Spectral theorem for a bounded selfadjoint linear operator.

**Normal Operators:**

Spectral properties for bounded normal operators, Spectral theorem for bounded normal operators.

### **Unbounded linear operators in Hilbert space:**

Hellinger-Toeplitz theorem, Symmetric and selfadjoint operators, Closed linear operators, Spectrum of an unbounded selfadjoint linear operator, Cayley Transformation  $U = (T - iI)(T + iI)^{-1}$  of an operator  $T$ , Spectral theorem for unitary and selfadjoint operators, Multiplication operator and differentiation operator, Application to Quantum Mechanics.

#### Reference Books :

6. Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley and sons.
7. G. Bachman and L. Narici, Functional Analysis, Dover Publications.
8. A. Taylor and D. Lay, Introduction to Functional Analysis, John Wiley and Sons.
9. N. Dunford and J.T. Schwartz, Linear Operators – 3, John Wiley and Sons.
10. P.R. Halmos, Introduction to Hilbert space and the theory of Spectral Multiplicity, Chelsea Publishing Co., N.Y.

### **B – 2.31: Plasma Mechanics II (50 Marks)**

Vlasov – Boltzmann self-consistent equations in collision less plasma. Plasma Oscillations. Landau. Damping

Waves in Plasmas : Waves in warm field free plasma : Dielectric Tensor and general dispersion relation – High frequency approximation & Low frequency approximation. Waves in cold homogeneous magneto plasma : Dielectric Tensor and general dispersion relation, Cut – Off & Resonance, Group Velocity, Wave normal surface, Refractive Index, Refractive Index Surface. Different types of wave modes for different types of approximation. Waves in warm homogeneous magneto plasma : Dielectric Tensor and general dispersion relation, Two fluid Model, Single fluid model, Two stream instability. Characterization of different waves in plasmas : Electron waves (electrostatic) : Plasma Oscillations, Upper hybrid Oscillations, Ion waves (electrostatic) : Ion-acoustic waves, Ion-cyclotron waves, Lower hybrid Oscillations, Electron waves (electromagnetic) : - Light waves, O waves, X waves, R waves (whistler mode), L Waves, Ion waves (electromagnetic) : - Alfvén wave, Magnetosonic wave

Nonlinear wave processes in plasma: Derivation of KdV-ZK equation for ion-acoustic wave & Alfvén wave and their soliton solution.

Applications: Space Plasma, Solar Plasma, Gravitational Plasma, Laboratory Plasma,

**References:**

3. Plasma Physics and controlled Fusion, F.F. Chen, PLENUM PRESS, NEWYORK AND LONDON.
4. Fundamental of Plasma Physics, J.A. Bittencourt, PERGAMON PRESS, NEWYORK AND LONDON.

Theory of Plasma waves, T.H. Stix, McGraw Hill.

**B –2. 32 : Probability and Stochastic Processes II (50 Marks)**

Markoff chain with stationary transition probabilities. Communication. Classification of states. Periodicity. Transient, null recurrent and positive recurrent states. Limiting probabilities. Ergodicity. Algebraic theory.

Continuous-parameter markoff chains. Simple Markoff processes. Poisson process Characteristic functional. Polya process. Birth-and-death process.

Branching process. Galton-Watson process. Continuous-parameter branching process. Age-dependent branchin process.

General theory of continuous-time processes. Kolmogoroff's forward and backward equations. Fokker-Planck equations. Wiener process.

**References:**

6. Bhat, B.R. – Modern Probability Theory.
7. Chung, K.L. – Elementary Probability Theory and Stochastic Processes.
8. Billingsley, P. – Probability and Measure.
9. Srinivasan, S.K. and Mehata, K.M. – Stochastic Processes.
10. Hoel, P.G.,Port, S.C. and Stone, C.J. – Introduction to Stochastic Processes.

**B –2. 33 : Production and Inventory Control II (50 marks)**

Probabilistic demand models. Expected cost. Probabilistic order level systems. Probabilistic order level systems with instantaneous demand. Probabilistic order level systems with uniform demand. Probabilistic order level systems with lead time. Discrete and continuous probability versions of the models. Problems on the two versions of the models. Newspaper boy problem. Spare parts problem. Baking company problem. Equivalence of probabilistic order level systems.

**References:**

Inventory Systems : Eliczer Nadder – John Wiley and Sons.

Analysis of Inventory Systems : G. Hadley and T. M. Whitin – Prentice Hall.

Principles of Inventory and Material Management : R. J. Tersine – North-Holland.

## **B- 2.34: Quantum Field Theory and Statistical Mechanics II (50 marks)**

### **Statistical Mechanics (50 marks)**

Kinetic theory of Gases : Introduction, Clausius' Ideas, Maxwell's law for velocity – distribution, Boltzmann's law of distribution. Boltzmann's Fundamental Kinetic equation, Boltzmann's H-theorem, Equilibrium distribution Zermelo's paradox based on Poincaré's Recurrence theorem, Ergodic Hypothesis, Ideal gas, Mean free-paths, Boltzmann's Transport equation and applications, virial equation of Clausius-read gas.

Statistical Mechanics after Gibbs : Introduction, some important concept of general mechanics, Gibbs ensemble, Liouville's theorem, canonical ensemble, fluctuation for canonical ensemble, Microcanonical ensemble, canonical distribution in mew-space, in configurational and momentum space, Grand ensemble and physical interpretation.

Statistical mechanics based on Boltzmann's hypothesis : Introduction. Maxwell-Boltzmann's statistics, system of linear oscillators and application, Bose-statistics for photons, Bose-Einstein ideal gas-phase condensation, Fermi-Dirac statistics, steady conduction of Heat and Electricity by Electrons in metal, Real gas – vanderwall equation of state, Cluster expansion, phase transtions.

### **References::**

1. The principle in statistical mechanics (Oxford Univ. Press) – R.C. Tolman.
2. Statistical Mechanics (Camb. Univ. Press) – R.H. Fowler.
3. The Kinetic Theory of gases (Dover series) – L.B. Loeb.
4. Elementary Principles in Statistical Mechanics (Yale Univ. Press) – J.W. Gibbs.
5. Statistical Mechanics (Wiley Eastern) – K. Huang.
6. Statistical Mechanics – J. Bhattacharya.

## **B – 2.35 : Queuing Theory and Game Theory II (50Marks)**

1. What is Game Theory? A game is a collection of four components:

players, strategies of players, outcomes and payoffs. We call games not only like chess, poker, bridge and so forth , but also conflicts between companies, military forces and nations.

### **2. Static vs. dynamic games**

### **3. Cooperative vs. non-cooperative games**

### **4. Related areas**

differential games, optimal control theory, mathematical economics

## 5. Application areas

corporate decision making, defense strategy, market modeling, public policy analysis, environmental systems, distributed computing, telecommunications networks

### References:

1. Introduction to Game Theory by Peter Morris, Springer
2. An Introduction to Game Theory by Martin J Osborne, Oxford University Press
3. Game Theory by L. A. Petrosjan, Nikolay A. Zenkevich, World Scientific

## B – 2.36: Epidemiology and Eco-epidemiology (50 Marks)

### Deterministic Epidemic Models:

Deterministic model of simple epidemic, Infection through vertical and horizontal transmission, General epidemic- Karmac-Mackendric Threshold Theorem, Recurrent epidemics, Seasonal variation in infection rate, allowance of incubation period, models with undamped waves, modeling of Venereal diseases, Simple model for the spatial spread of an epidemic.

### Non Constant Total Population Model in Epidemic:

Introduction, Parasite-host system, an SIS model, an SIR model and an SIRS model.

### Stochastic Epidemic Models:

Introduction, stochastic simple epidemic model, Yule-Furry model (pure birth process), expectation and variance of infective, calculation of expectation by using moment generating function.

### Proportional Mixing Rate in Epidemic:

Introduction, SIS model with proportional mixing rate, SIRS model with proportional mixing rate.

### Eco-Epidemiology:

Introduction, host-parasite-predator systems, viral infection on phytoplankton-zooplankton (prey-predator) system.

### References:

1. N.T.J.Bailey (1975): The Mathematical Theory of Infectious Diseases and its Application, 2nd edn. London, Griffin .
2. J.D.Murray (1990): Mathematical Biology, Springer and Verlag.
3. Vincenzo Capasso (1993): Lecture Notes in Mathematical Biology (Vol. No. 97)- Mathematical Structures of Epidemic Systems, Springer Verlag.
4. Eric Renshaw (1990): Modelling Biological Populations in Space and Time, Cambridge Univ. Press.
5. Busenberg and Cooke (1993): Vertically Transmitted Diseases- Models and Dynamics, Springer Verlag.

## **B –2. 37 : Theory of Marketing Decisions II (50 marks)**

Marketing management function, Sequence of marketing decisions, Marketing models-construction and types. Certainty and the marketing decision, Risk and the marketing decision, Uncertainty and the marketing decision, Criterion of pessimism, Criterion of optimism, Criterion of regret, Laplace criterion, Hurwicz criterion, Decision tree analysis.

Application of Bayesian decision theory in marketing management, Subjective and objective probability, Delphi process of pooling group opinion, Prior analysis, Posterior analysis, Pre- posterior analysis, Comparison of prior, posterior and pre-posterior analysis, Applications.

Advertising decisions, Determination of advertising goals – exposure, awareness and attitude, Response of sales to advertising, Vidale–Wolfe model, Determination of advertising budget, Monopoly advertising under static and dynamic conditions, Competitive strategy of promotion, Competitive models under uncertainty – maximin criterion, minimax criterion, Hurwicz criterion, Laplace criterion, minimax regret criterion.

Consumer buying behaviour, Warranty reserve, Competitive bidding model.

### **References:**

1. Mathematical Analysis for Economists – R. G. D. Allen : McMillan.
2. Mathematical Economics – R. G. D. Allen : McMillan.
3. Mathematical Formulation of Micro-economists – M. M. Metwally: Asia Publishing House.
4. Quantitative Techniques for Marketing Decisions – Marvin A. Jolson and Richard T. Hisc : McMillan.
5. Marketing Management – Philip Kotler : Prentice Hall.

## **B – 2.38: Theory of Semigroups II (50 Marks)**

**0-Simple Semigroups:** Simple and 0-simple semigroups. Principal factors, Completely simple, completely 0-simple semigroups. Rees' theorem. Primitive idempotents. (20)

**Bands:** Bands, Free bands, Varieties of bands. (10)

**Inverse semigroups:** Definition, Elementary properties of inverse semigroups, Congruences on inverse semigroups, Fundamental inverse semigroups. (10)

**Automata Theory:** Free Monoids, Languages and Codes, Automata, Rational Languages and their Syntactic Monoids. (10)

### **References:**

1. Fundamentals of semigroup theory, J. M. Howie, Clarendon Press, oxford, 1995.
2. An introduction to semigroup theory, J. M. Howie, Academic Press, London, 1976.



3. The algebraic theory of semigroups, Amer. Math. Soc., Math Surveys No. 7, Providence, Vol I, 1961, Vol II, 1967.
4. Completely Regular Semigroups, M. Petrich and N. R. Reilly, John Wiley & Sons.
5. Introduction to semigroups, M. Petrich, Merrill, Columbus, 1973.
6. Structure of regular semigroups, M. Petrich, Univ. de Montpellier, 1977.
7. Lectures in semigroups, M. Petrich,
8. Semigroups and combinatorial applications, G. Lallement,
9. Completely 0-simple semigroups: an abstract treatment of the lattice of congruences, Benjamin, New York, 1969.
10. A course in universal algebra, S. Burris and H. P. Sankappanavar, Springer, New York, 1981.

### **B – 2.39 : Topological Groups and Harmonic Analysis II ( 50 Marks)**

The Haar Integral, Haar Measure on Locally Compact Groups, Convolutions of Functions and Measures (Discussions without proof).

Haar Measure on  $\mathbb{R}$ ,  $\mathbb{T}$ ,  $\mathbb{Z}$  and some simple matrix groups, Approximate Identities, Fourier Series, Fejer's Theorem, The Classical Kernels, Fejer's, Poisson's and Dirichlet's Summability in Norm and Pointwise Summability, Fatou's Theorem, The Fourier Transform, Kernels on  $\mathbb{R}$ , The Plancherel Theorem on  $\mathbb{R}$ , Plancherel Measure on  $\mathbb{R}$ .

Elements of Representation Theory, Unitary Representations of Locally Compact Groups.

#### **References:**

1. Hewitt, E., Ross, K., *Abstract Harmonic Analysis*, Vol I, Academic Press, NY, 1963.
2. Bachman, G., *Elements of Abstract Harmonic Analysis*, Academic Press, NY and London, 1964.
3. Rudin, W., *Fourier Analysis on Groups*, McGraw Hill Publishing Co. Ltd.
4. Loomis, L., *An Introduction to Abstract Harmonic Analysis*, Van Nostrand, N.J., 1953.
5. Goldberg, R.R., *Fourier Transforms*, Cambridge University Press, London & NY, 1961.